

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTIONS 2.8 & 2.9

Math 2000 Worksheet

WINTER 2020

For practice only. Not to be submitted.

1. Evaluate the following double integrals by rewriting them in polar coordinates.

(a)
$$\int_{-4}^4 \int_0^{\sqrt{16-y^2}} \sqrt{x^2 + y^2 + 9} \, dx \, dy$$

(b)
$$\iint_D \sqrt{x^2 + y^2} \, dA$$
 where D is the region inside the circle $x^2 + (y - 1)^2 = 1$

(c)
$$\iint_D \frac{y^2}{x^2} \, dA$$
 where D is the part of the annulus (ring) $9 \leq x^2 + y^2 \leq 25$ lying in the first quadrant and below the line $y = x$

2. Use a double integral in either Cartesian or polar coordinates to find the volume of each solid.

(a) the solid bounded above by the curve $f(x, y) = 1 - xy$, below by the xy -plane, and whose cross-section is the region bounded by the curves $y = x$ and $y = x^2$

(b) the solid under the paraboloid $z = 3x^2 + y^2$, above the xy -plane, and whose cross-section is the region bounded by the curves $y = x$ and $x = y^2 - y$

(c) the solid bounded by the surface $z = xy$, the cylinders $y = x^2$ and $x = y^2$, and the plane $z = 0$

(d) the solid that lies under the cone $z = \sqrt{x^2 + y^2}$, above the xy -plane, and whose cross section is the annulus $4 \leq x^2 + y^2 \leq 25$

(e) the solid under the surface $z = 1 + xy$, above the xy -plane, and whose cross-section is the triangle with vertices $(1, 1)$, $(4, 1)$ and $(3, 2)$

(f) the solid bounded by the paraboloid $z = 4 - x^2 - y^2$ and the xy -plane