MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.3 | **Math 2000 Worksheet** | WINTER 2020

For practice only. Not to be submitted.

1. Find all the first-order partial derivatives for each of the following functions.

   (a) $z = \sin(x)\cos(y)$

   (b) $f(x, y) = y^x$

   (c) $f(s, t) = \arctan\left(\dfrac{s^2}{t^2}\right)$

   (d) $z = \cos(3x - 5y)$ at the point $\left(\dfrac{\pi}{6}, \dfrac{\pi}{15}, \dfrac{\sqrt{2}}{2}\right)$

   (e) $w = xy^2z^3$

   (f) $f(x, y, z) = \dfrac{6xy}{\sqrt{25 - z^2}}$ at the point $(1, 2, 4, 4)$.

2. Find all the second-order partial derivatives of $f(x, y) = xye^y$. Does Clairault's Theorem hold for this function?

3. Consider the function $z = e^{3x}\sin(5y)$. Find the third-order partial derivatives z_{xxy}, z_{yxx}, z_{yxy} and z_{yyx}. Are your results consistent with Clairault's Theorem?

4. Determine which of the following functions are solutions of Laplace's equation.

   (a) $f(x, y) = x^2 - y^2$

   (b) $f(x, y) = x^2 + y^2$

   (c) $f(x, y) = \ln[(x^2 + y^2)^2]$

   (d) $f(x, y) = e^{-x}\cos(y) - e^{-y}\cos(x)$

5. Show that $u = \sin(kx)\sin(\alpha kt)$ is a solution of the **wave equation**

$$\frac{\partial^2 u}{\partial t^2} = \alpha^2 \frac{\partial^2 u}{\partial x^2},$$

   where α and k are constants.

6. Show that $f(x, y) = xe^y + ye^x$ is a solution of the partial differential equation

$$f_{xxx}(x, y) + f_{yyy}(x, y) = xf_{xyy}(x, y) + yf_{xxy}(x, y).$$