

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.2

Math 2000 Worksheet

WINTER 2020

For practice only. Not to be submitted.

1. Use the basic properties of convergent sequences and results regarding limits of common sequences to evaluate the limit, if it exists, of each of the following sequences $\{a_i\}$. If a sequence is divergent, explain why.

$$\begin{array}{lll} \text{(a)} & a_i = \frac{\sqrt{i}}{2 - \sqrt{i}} & \text{(b)} & a_i = \frac{i}{2 - \sqrt{i}} & \text{(c)} & a_i = 7 - \left(-\frac{1}{4}\right)^i \\ \text{(d)} & a_i = \frac{3 \cdot 7^i}{2^{3i-1}} & \text{(e)} & a_i = \frac{5^i + 1}{5^i - 1} & \text{(f)} & a_i = \frac{5^i + 1}{3^i - 2^i} \end{array}$$

2. Use any appropriate method to evaluate the limit, if it exists, of each of the following sequences $\{a_i\}$. If a sequence is divergent, explain why.

$$\begin{array}{lll} \text{(a)} & a_i = 1 + \sin\left(\frac{i\pi}{2}\right) & \text{(b)} & a_i = \frac{i!}{(i+2)!} & \text{(c)} & a_i = \frac{1}{i^2} + \frac{2}{i^2} + \cdots + \frac{i}{i^2} \\ \text{(d)} & a_i = \frac{\sin^2(i)}{5^i} & \text{(e)} & a_i = \frac{\ln(2 + e^i)}{9i} & \text{(f)} & a_i = \left(1 + \frac{3}{i}\right)^i \end{array}$$

3. Determine whether each of the following sequences $\{a_i\}$ is increasing, decreasing or not monotonic. If it is not monotonic, is there a tail of the sequence which is increasing or decreasing? Finally, is $\{a_i\}$ bounded?

$$\begin{array}{l} \text{(a)} & a_i = \frac{3i - 7}{4i + 1} \\ \text{(b)} & a_i = \cos\left(\frac{i\pi}{3}\right) \\ \text{(c)} & a_i = \frac{4\sqrt{i}}{i + 5} \\ \text{(d)} & a_i = \frac{1 \cdot 4 \cdot 7 \cdots (3i - 2)}{3 \cdot 6 \cdot 9 \cdots (3i)} \end{array}$$