## MEMORIAL UNIVERSITY OF NEWFOUNDLAND DEPARTMENT OF MATHEMATICS AND STATISTICS

## TEST 1

## MATHEMATICS 2000

February 12th, 2020

## Name

**MUN Number** 

1. Find the limit of each sequence  $\{a_i\}$ , or explain why it diverges. [13]

(a) 
$$a_i = \frac{2i^3(i-5)}{(3i^2-4)(3i^2+4)}$$
  
(b)  $a_i = \frac{\sin^2(i)}{5^i}$   
(c)  $a_i = \frac{\ln(\ln(i))}{\sqrt{i}}$ 

2. Based on your results for #1(a), can anything be determined about whether the series  $\sum_{i=1}^{\infty} \frac{2i^3(i-5)}{(3i^2-4)(3i^2+4)}$  converges or diverges? Justify your answer. [3]

[6] 3. Use the Bounded Monotonic Sequence Theorem to show that  $\{a_i\}$  is convergent, where

$$a_i = \frac{4(i-1)!}{1 \cdot 5 \cdot 9 \cdots (4i-3)}.$$

[4] 4. Find the sum of the convergent series  $\sum_{i=0}^{\infty} \frac{3^i - 4^i}{3^i 4^i}$ .

[4] 5. Find and sketch the domain of the function  $f(x,y) = \frac{\sqrt{y-x}}{x+3}$ .

[5] 6. Evaluate  $\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3}$  or show that the limit does not exist.

[5] 7. Find all the first- and second-order partial derivatives of  $z = x \ln(x) \sin(5y)$ .