MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.2 **Math 2000 Worksheet** WINTER 2020

SOLUTIONS

1. (a) Since this function is a polynomial, we can simply use direct substitution:

$$\begin{aligned}\lim_{(x,y)\to(3,-1)} x^2y^3+5xy-6y+9 &= 3^2(-1)^3+5(3)(-1)-4(-1)+9\\ &= -9-15+4+9\\ &= -11.\end{aligned}$$

(b) We try direct substitution and see that

$$\lim_{(x,y)\to(2,5)} \frac{xy-8x-2y+4}{y^2-y-2} = \frac{10-16-10+4}{25-5-2} = \frac{-12}{18} = -\frac{2}{3}.$$

(c) Direct substitution results in a $\frac{0}{0}$ indeterminate form, but we can factor and cancel:

$$\lim_{(x,y)\to(2,5)} \frac{xy-4x-2y+8}{x^2-x-2} = \lim_{(x,y)\to(2,5)} \frac{(x-2)(y-4)}{(x-2)(x+1)} = \lim_{(x,y)\to(2,5)} \frac{y-4}{x+1} = \frac{1}{3}.$$

(d) First we let $(x,y)\to(0,0)$ along the line $y=0$ so the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{4x^2+\sin^2(y)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{4x^2}{x^2} = \lim_{(x,y)\to(0,0)} 4 = 4.$$

Next let $(x,y)\to(0,0)$ along the line $x=0$ so the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{4x^2+\sin^2(y)}{x^2+y^2} = \lim_{y\to0} \frac{\sin^2(y)}{y^2} = \lim_{y\to0}\left(\frac{\sin(y)}{y}\right)^2 = 1^2 = 1.$$

Since these values differ, the limit does not exist.

(e) First we let $(x,y)\to(0,0)$ along the line $y=0$ so the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{x\to0}\frac{0}{x^6} = \lim_{x\to0} 0 = 0.$$

Next we could try letting $(x,y)\to(0,0)$ along the line $x=0$, but then we obtain

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{y\to0}\frac{0}{3y^3} = \lim_{x\to0} 0 = 0,$$

which is the same value we have already computed. Similarly, if we use the path $y=x$, we get

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{x\to0}\frac{12x^4}{x^6+3x^3} = \lim_{x\to0}\frac{12x}{x^3+3} = 0.$$

But if we consider the path $y=x^2$, we have

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{x\to0}\frac{12x^6}{4x^6} = \lim_{x\to0} 3 = 3,$$

and since this differs from the previous values, we conclude that the limit does not exist.

2. We immediately have $f(3,-2)=0$. Next,

$$\begin{aligned}\lim_{(x,y)\to(3,-2)} f(x,y) = \lim_{(x,y)\to(3,-2)} \frac{4x^2+12xy+9y^2}{4x^2-9y^2} &= \lim_{(x,y)\to(3,-2)} \frac{(2x+3y)(2x+3y)}{(2x-3y)(2x+3y)} \\ &= \lim_{(x,y)\to(3,-2)} \frac{2x+3y}{2x-3y} \\ &= \frac{0}{12} \\ &= 0.\end{aligned}$$

Hence the limit exists, and furthermore we can now see that

$$f(3,-2) = \lim_{(x,y)\to(3,-2)} f(x,y).$$

Thus $f(x,y)$ is continuous at $(x,y)=(3,-2)$.