# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.1 **Math 2000 Worksheet** WINTER 2020

## SOLUTIONS

1. (a) We can write

$$\frac{(2i)!}{2\cdot 4\cdot 6\cdots 2i} = \frac{1\cdot 2\cdot 3\cdot 4\cdots(2i-1)(2i)}{2\cdot 4\cdot 6\cdots 2i} = 1\cdot 3\cdot 5\cdots(2i-1).$$

(b) Here,

$$\frac{2\cdot 4\cdot 6\cdots(2i)}{5\cdot 10\cdot 15\cdots(5i)} = \frac{2^i(1\cdot 2\cdot 3\cdots i)}{5^i(1\cdot 2\cdot 3\cdots i)} = \frac{2^i i!}{5^i i!} = \left(\frac{2}{5}\right)^i.$$

2. (a) $\{a_i\} = \left\{1, 0, -\frac{1}{9}, 0, \frac{1}{25}, \ldots\right\}$

(b) $\{a_i\} = \left\{2, -\frac{3}{2}, \frac{2}{3}, -\frac{5}{24}, \frac{1}{20}, \ldots\right\}$

(c) $\{a_i\} = \left\{4, \frac{2}{3}, \frac{1}{4}, \frac{1}{9}, \frac{1}{19}, \ldots\right\}$

3. (a) $\{a_i\} - \{b_i\} = \{2i - 2^i\} = \{0, 0, -2, -8, -22, \ldots\}$

(b) $4\{b_i\} = \{4\cdot 2^i\} = \{2^{i+2}\} = \{8, 16, 32, 64, 128, \ldots\}$

(c) $\{a_i\}\cdot\{b_i\} = \{2i\cdot 2^i\} = \{i2^{i+1}\} = \{4, 16, 48, 128, 320, \ldots\}$

(d) $\dfrac{\{b_i\}}{\{a_i\}} = \left\{\dfrac{2^i}{2i}\right\} = \left\{\dfrac{2^{i-1}}{i}\right\} = \left\{1, 1, \dfrac{4}{3}, 2, \dfrac{16}{5}, \ldots\right\}$

4. (a) The numerator of each term is simply $i$, while the denominator in each case is a perfect cube, starting with $2^3$. Thus

$$a_i = \frac{i}{(i+1)^3}.$$

(b) This is an alternating sequence, so let's first consider the sequence consisting of the absolute value of each term,

$$\{p_i\} = \{3, 8, 13, 18, \ldots\}.$$

The difference between each term is 5, so

$$p_i = 5i - 2.$$

Thus, for the original sequence,

$$a_i = (-1)^i(5i-2).$$

(c) It's obvious that the sequence is formed by alternately subtracting or adding 8 to the preceding term. However, this seems difficult to represent as an equation. It's easier to view this as an alternating sequence in which 4 is either added to or subtracted from 6. Thus

$$a_i = 6 + 4(-1)^{i+1}.$$