MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.2 | **Math 2000 Worksheet** | FALL 2018

SOLUTIONS

1. (a) Since this function is a polynomial, we can simply use direct substitution:

$$\lim_{(x,y)\to(3,-1)} x^2y^3+5xy-6y+9 = 3^2(-1)^3+5(3)(-1)-4(-1)+9 = -9-15+4+9 = -11.$$

(b) Direct substitution results in a $\frac{0}{0}$ indeterminate form, but we can factor and cancel:

$$\lim_{(x,y)\to(2,5)} \frac{xy-4x-2y+8}{x^2-x-2} = \lim_{(x,y)\to(2,5)} \frac{(x-2)(y-4)}{(x-2)(x+1)} = \lim_{(x,y)\to(2,5)} \frac{y-4}{x+1} = \frac{1}{3}.$$

(c) First we let $(x,y)\to(0,0)$ along the line $y=0$ so the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{4x^2+\sin^2(y)}{x^2+y^2} = \lim_{(x,y)\to(0,0)} \frac{4x^2}{x^2} = \lim_{(x,y)\to(0,0)} 4 = 4.$$

Next let $(x,y)\to(0,0)$ along the line $x=0$ so the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{4x^2+\sin^2(y)}{x^2+y^2} = \lim_{y\to0} \frac{\sin^2(y)}{y^2} = \lim_{y\to0}\left(\frac{\sin(y)}{y}\right)^2 = 1^2 = 1.$$

Since these values differ, the limit does not exist.

(d) First we let $(x,y)\to(0,0)$ along the line $y=0$ so the limit becomes

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{x\to0}\frac{0}{x^6} = \lim_{x\to0} 0 = 0.$$

Next we could try letting $(x,y)\to(0,0)$ along the line $x=0$, but then we obtain

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{y\to0}\frac{0}{3y^3} = \lim_{x\to0} 0 = 0,$$

which is the same value we have already computed. Similarly, if we use the path $y=x$, we get

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{x\to0}\frac{12x^4}{x^6+3x^3} = \lim_{x\to0}\frac{12x}{x^3+3} = 0.$$

But if we consider the path $y=x^2$, we have

$$\lim_{(x,y)\to(0,0)} \frac{12x^4y}{x^6+3y^3} = \lim_{x\to0}\frac{12x^6}{4x^6} = \lim_{x\to0} 3 = 3,$$

and since this differs from the previous values, we conclude that the limit does not exist.