MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 1 | **MATH 2000** | Fall 2018

SOLUTIONS

[4] 1. (a) We can write

$$\frac{i!}{(i+3)!} = \frac{1\cdot 2\cdot 3\cdots i}{1\cdot 2\cdot 3\cdots i(i+1)(i+2)(i+3)} = \frac{1}{(i+1)(i+2)(i+3)} = \frac{1}{i^3+6i^2+11i+6}.$$

[4] (b) We can write

$$\frac{(4i)!}{(3i)!} = \frac{1\cdot 2\cdot 3\cdots(3i)(3i+1)(3i+2)\cdots(4i)}{1\cdot 2\cdot 3\cdots(3i)} = (3i+1)(3i+2)\cdots(4i).$$

[4] (c) Both the numerator and the denominator are products of odd integers. However, the smallest factor in the numerator is 7 and the largest is $2i+5$, while the smallest factor in the denominator is 1 and the largest is $2i-1$, so not every factor cancels:

$$\begin{aligned}\frac{7\cdot 9\cdot 11\cdots(2i+5)}{1\cdot 3\cdot 5\cdots(2i-1)} &= \frac{7\cdot 9\cdot 11\cdots(2i-1)(2i+1)(2i+3)(2i+5)}{1\cdot 3\cdot 5\cdot 7\cdot 9\cdot 11\cdots(2i-1)}\\ &= \frac{(2i+1)(2i+3)(2i+5)}{1\cdot 3\cdot 5}\\ &= \frac{(2i+1)(2i+3)(2i+5)}{15}\\ &= \frac{8i^3+36i^2+46i+15}{15}.\end{aligned}$$

[2] 2. (a) According to the formula, we have

$$\begin{aligned}a_1 &= (-1)\frac{1^2+1}{1^2+1} = -1\\ a_2 &= (-1)^2\frac{2^2+1}{2^2+2} = \frac{5}{6}\\ a_3 &= (-1)^3\frac{3^2+1}{3^2+3} = -\frac{5}{6}\\ a_4 &= (-1)^4\frac{4^2+1}{4^2+4} = \frac{17}{20}\\ a_5 &= (-1)^5\frac{5^2+1}{5^2+5} = -\frac{13}{15}\end{aligned}$$

so

$$\{a_i\} = \left\{-1, \frac{5}{6}, -\frac{5}{6}, \frac{17}{20}, -\frac{13}{15}, \ldots\right\}.$$

[2] (b) According to the formula, we have

$$\begin{aligned}
a_1 &= \frac{1}{1!} = \frac{1}{1} = 1 \\
a_2 &= \frac{1 \cdot 4}{2!} = \frac{4}{2} = 2 \\
a_3 &= \frac{1 \cdot 4 \cdot 7}{3!} = \frac{28}{6} = \frac{14}{3} \\
a_4 &= \frac{1 \cdot 4 \cdot 7 \cdot 10}{4!} = \frac{280}{24} = \frac{35}{3} \\
a_5 &= \frac{1 \cdot 4 \cdot 7 \cdot 10 \cdot 13}{5!} = \frac{3640}{120} = \frac{91}{3},
\end{aligned}$$

so

$$\{a_i\} = \left\{1, 2, \frac{14}{3}, \frac{35}{3}, \frac{91}{3}, \dots\right\}.$$

[2] 3. Using the recursion formula, we have

$$\begin{aligned}
a_3 &= \frac{a_1 - 1}{a_2 + 1} = \frac{-3 - 1}{1 + 1} = -2 \\
a_4 &= \frac{a_2 - 1}{a_3 + 1} = \frac{1 - 1}{-2 + 1} = 0 \\
a_5 &= \frac{a_3 - 1}{a_4 + 1} = \frac{-2 - 1}{0 + 1} = -3 \\
a_6 &= \frac{a_4 - 1}{a_5 + 1} = \frac{0 - 1}{-3 + 1} = \frac{1}{2} \\
a_7 &= \frac{a_5 - 1}{a_6 + 1} = \frac{-3 - 1}{\frac{1}{2} + 1} = -\frac{8}{3}
\end{aligned}$$

so

$$\{a_i\} = \left\{-3, 1, -2, 0, -3, \frac{1}{2}, -\frac{8}{3}, \dots\right\}.$$

[4] 4. (a) This is an alternating sequence, so we know that a_i will take the form

$$a_i = (-1)^{i+1} p_i$$

where

$$\{p_i\} = \{6, 24, 120, 720, 5040, \dots\}.$$

Turning our attention to $\{p_i\}$, then, we can see that each of these terms is a factorial, starting with $p_1 = 3!$. Thus $p_i = (i + 2)!$ and so

$$a_i = (-1)^{i+1}(i + 2)!.$$

[4] (b) We can recognise the numerators of each term as powers of 2, beginning with 2^0. The denominators, similarly, are powers of 3, starting with 3^2. Thus

$$a_i = \frac{2^{i-1}}{3^{i+1}}.$$

[4] (c) Observe that the third and fifth terms have denominators which are perfect squares (that is, 3^2 and 5^2), and that this is also trivially true of the first term. So let's try rewriting all of the terms in this form, obtaining

$$\left\{ \frac{7}{1^2}, \frac{12}{2^2}, \frac{17}{3^2}, \frac{22}{4^2}, \frac{27}{5^2}, \ldots \right\}.$$

Now we can see that the terms in the numerator are increasing by 5, and in fact are always 2 greater than the nearest multiple of 5. Thus we deduce that

$$a_i = \frac{5i+2}{i^2}.$$

[5] 5. (a) For the numerator to be defined, we must have $y - x^2 \geq 0$, so $y \geq x^2$. To ensure that there is no division by zero, we must have $x^2 - 4 \neq 0$, that is, $x \neq \pm 2$. Hence the domain is the parabola $y = x^2$ and its interior, except those points which lie on the vertical lines $x = 2$ and $x = -2$.

[5] (b) In the numerator, we require $x > 0$. In the denominator, we must have $y^2 - x^2 \neq 0$. The corresponding equation is

$$y^2 - x^2 = 0 \quad \Longrightarrow \quad (y-x)(y+x) = 0$$

so $y = x$ or $y = -x$. Hence the domain consists of the first and fourth quadrants of the xy-plane, except for those points which lie on the lines $y = x$ and $y = -x$.