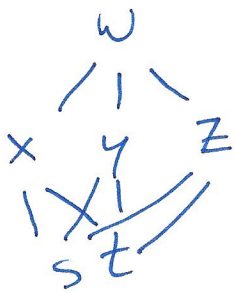


9. a)



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial s} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial s} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial w}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial w}{\partial z} \cdot \frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial x} = 2x \quad \frac{\partial w}{\partial y} = -2y \quad \frac{\partial w}{\partial z} = 2z \quad \frac{\partial x}{\partial s} = 2s \quad \frac{\partial y}{\partial s} = -2s$$

$$\frac{\partial z}{\partial s} = t \quad \frac{\partial x}{\partial t} = 2t \quad \frac{\partial y}{\partial t} = 2t \quad \frac{\partial z}{\partial t} = s$$

$$\begin{aligned} \text{So } \frac{\partial w}{\partial s} &= 2x \cdot 2s + (-2y) \cdot (-2s) + 2z \cdot t \\ &= 4xs + 4ys + 2zt \end{aligned}$$

$$\begin{aligned} \frac{\partial w}{\partial t} &= 2x \cdot 2t + (-2y) \cdot (2t) + (2z) \cdot s \\ &= 4xt - 4yt + 2zs \end{aligned}$$

b) We define $F(x, y, z) = xe^y + y^2 \ln(x) + z^2 y - 8z$

$$F_x(x, y, z) = e^y + \frac{y^2}{x}$$

$$F_y(x, y, z) = xe^y + 2y \ln(x) + z^2$$

$$F_z(x, y, z) = 2zy - 8$$

$$\text{Then } \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{e^y + \frac{y^2}{x}}{2zy - 8} = -\frac{xe^y + y^2}{2zyx - 8x}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{xe^y + 2y \ln(x) + z^2}{2zy - 8}$$