6. $k_i = \dfrac{i}{5^i(i^2+2)}$ $\qquad k_{i+1} = \dfrac{i+1}{5^{i+1}[(i+1)^2+2]}$

$$\rho = \lim_{i\to\infty} \left|\frac{k_{i+1}}{k_i}\right|$$

$$= \lim_{i\to\infty} \frac{i+1}{5^{i+1}[(i+1)^2+2]} \cdot \frac{5^i(i^2+2)}{i}$$

$$= \lim_{i\to\infty} \frac{(i+1)(i^2+2)}{5i(i^2+2i+3)}$$

$$= \lim_{i\to\infty} \frac{i^3+i^2+2i+2}{5i^3+10i^2+15i} = \frac{1}{5} \rightarrow R = 5$$

The power series converges for at least

$$|x+3| < 5$$

$$-5 < x+3 < 5$$

$$-8 < x < 2$$

At $x=2$, the power series becomes $\displaystyle\sum_{i=1}^{\infty} \frac{i}{5^i(i^2+2)} \cdot 5^i = \sum_{i=1}^{\infty} \frac{i}{i^2+2}$

We test against $\sum t_i = \sum \frac{i}{i^2} = \sum \frac{1}{i}$ (divergent p-series). We have

$$\lim_{i\to\infty} \frac{a_i}{t_i} = \lim_{i\to\infty} \frac{i}{i^2+2} \cdot i = \lim_{i\to\infty} \frac{i^2}{i^2+2} = 1$$

By the Limit Comparison Test, the series is divergent.

At $x=-8$, the power series becomes $\displaystyle\sum_{i=1}^{\infty} \frac{i}{5^i(i^2+2)} (-5)^i = \sum_{i=1}^{\infty} \frac{(-1)^i i}{i^2+2}$

We have $p_i = \dfrac{i}{i^2+2}$ so $\displaystyle\lim_{i\to\infty} p_i = \lim_{i\to\infty} \frac{i}{i^2+2} = 0$. Also if

$$f(x) = \frac{x}{x^2+2} \rightarrow f'(x) = \frac{(x^2+2)-2x^2}{(x^2+2)^2} = \frac{2-x^2}{(x^2+2)^2} < 0$$

so $\{p_i\}$ is decreasing. By the Alternating Series Test, the series is convergent.

The interval of convergence is $\underline{\underline{-8 \le x < 2}}$.