

5. The absolute series is $\sum_{i=1}^{\infty} \left| \frac{(-1)^i}{4i+3} \right| = \sum_{i=1}^{\infty} \frac{1}{4i+3}$.

We use the Limit Comparison Test with test series $\sum t_i = \sum \frac{1}{i}$ which is a divergent p -series. We get

$$\begin{aligned} \lim_{i \rightarrow \infty} \frac{|a_i|}{t_i} &= \lim_{i \rightarrow \infty} \frac{1}{4i+3} \cdot i \\ &= \lim_{i \rightarrow \infty} \frac{i}{4i+3} \\ &= \frac{1}{4} \end{aligned}$$

By the LCT, the absolute series is divergent. Hence the given series is not absolutely convergent.

To test the given series, we try the Alternating Series Test.

The positive part is $p_i = \frac{1}{4i+3}$. We have

$$\lim_{i \rightarrow \infty} p_i = \lim_{i \rightarrow \infty} \frac{1}{4i+3} = 0.$$

We let $f(x) = \frac{1}{4x+3}$ so $f'(x) = \frac{-4}{(4x+3)^2} < 0$ so

$\{p_i\}$ is decreasing. Hence by the AST, the given series is convergent.

We conclude that the given series is conditionally convergent.