

1. We consider the function

$$f(x) = \left(1 - \frac{5}{x}\right)^x$$

and note that $\lim_{x \rightarrow \infty} f(x) = 1^\infty$ indeterminate form, so we apply L'Hopital's Rule. We let

$$y = \ln(f(x)) = \ln\left(\left(1 - \frac{5}{x}\right)^x\right)$$

$$= x \ln\left(1 - \frac{5}{x}\right)$$

$$= \frac{\ln\left(1 - \frac{5}{x}\right)}{\frac{1}{x}}$$

so $\lim_{x \rightarrow \infty} y = \frac{0}{0}$ indeterminate form

$$\lim_{x \rightarrow \infty} y \stackrel{(\#)}{=} \lim_{x \rightarrow \infty} \frac{[\ln(1 - \frac{5}{x})]'}{[\frac{1}{x}]'}$$

$$= \lim_{x \rightarrow \infty} \frac{[\ln(\frac{x-5}{x})]'}{[\frac{1}{x}]'}$$

$$= \lim_{x \rightarrow \infty} \frac{\frac{x}{x-5} \cdot \frac{5}{x^2}}{-\frac{1}{x^2}}$$

$$= \lim_{x \rightarrow \infty} \frac{-5x}{x-5} = -\frac{5}{1} = -5$$

Therefore $\lim_{x \rightarrow \infty} f(x) = e^{-5}$ so, by the Evaluation Theorem,

$$\boxed{\lim_{i \rightarrow \infty} a_i = e^{-5}}$$