Section 1.6

If the denominator of the expression is of the form

$$k\sqrt{b} + l\sqrt{c}$$

then we multiply the numerator and the denominator by the conjugate of the denominator, which has the form

$$k\sqrt{b} - l\sqrt{c}.$$

Here we can use the special product

$$(B+C)(B-C) = B^2 - C^2$$

Thus the denominator becomes

$$(k\sqrt{b}+l\sqrt{c})(k\sqrt{b}-l\sqrt{c})$$

$$= (k\sqrt{b})^2 - (l\sqrt{c})^2$$

$$= k^2 b - l^2 c$$

eg
$$\frac{6}{3\sqrt{5}+2\sqrt{3}} = \frac{6}{3\sqrt{5}+2\sqrt{3}} \cdot \frac{3\sqrt{5}-2\sqrt{3}}{3\sqrt{5}-2\sqrt{3}}$$

$$= \frac{6(3\sqrt{5}-2\sqrt{3})}{(3\sqrt{5})^2-(2\sqrt{3})^2}$$

$$= \frac{6(3\sqrt{5}-2\sqrt{3})}{9\cdot 5 - 4\cdot 3}$$

$$= \frac{6(3\sqrt{5}-2\sqrt{3})}{33} \boxed{= \frac{2(3\sqrt{5}-2\sqrt{3})}{11}}$$

$$\text{eg} \quad \frac{2}{3-4\sqrt{x}} = \frac{2}{3-4\sqrt{x}} \cdot \frac{3+4\sqrt{x}}{3+4\sqrt{x}}$$

$$= \frac{2(3+4\sqrt{x})}{3^2-(4\sqrt{x})^2}$$

$$\boxed{= \frac{2(3+4\sqrt{x})}{9-16x}}$$

Section 1.7: Solving Equations

Def'n: An equation is a mathematical statement in which two expressions are set equal to each other. If the equation includes a variable, a solution of the equation is a value of the variable that makes the equation true. To solve the equation is to find all solutions.

Def'n: A linear equation is an equation that can be written in the form

$$ax + b = 0$$

for real numbers a, b with $a \neq 0$.

eg $4x - 6 = 0$ is a linear equation

To solve a linear equation, we just isolate x:

$$ax + b = 0$$

$$ax = -b$$

$$\frac{ax}{a} = -\frac{b}{a}$$

$$x = -\frac{b}{a}$$

eg Solve $4x-6=0$.

We have
$$4x-6=0$$
$$4x=6$$
$$\frac{4x}{4}=\frac{6}{4}$$
$$\boxed{x=\frac{3}{2}}$$

We can check our solutions by substituting back into the equation.

eg Here, we see that for $x=\frac{3}{2}$,
$$4x-6=4\cdot\frac{3}{2}-6=6-6=0.$$

eg Solve $\frac{x}{6}=1-\frac{5x}{9}$

We can rewrite this equation as
$$\frac{x}{6}+\frac{5x}{9}=1$$
$$\frac{x}{6}\cdot\frac{3}{3}+\frac{5x}{9}\cdot\frac{2}{2}=1$$
$$\frac{3x}{18}+\frac{10x}{18}=1$$
$$\frac{13x}{18}=1$$
$$\frac{13x}{18}\cdot\frac{18}{13}=1\cdot\frac{18}{13}$$
$$\boxed{x=\frac{18}{13}}$$

Def'n: A quadratic equation is an equation that can be written in the form

$$ax^2 + bx + c = 0$$

for real numbers a, b, c with $a \neq 0$.

eg $3x^2 - 9 = 0$ and $x^2 - 6x + 8 = 0$

are both quadratic equations

The easiest type of quadratic equation to solve is one in which $b = 0$ so we have only $ax^2 + c = 0$. They can be solved in a manner similar to linear equations.

eg Solve $3x^2 - 9 = 0$.

We have

$$3x^2 - 9 = 0$$

$$3x^2 = 9$$

$$\frac{3x^2}{3} = \frac{9}{3}$$

$$x^2 = 3$$

$$\boxed{x = \pm\sqrt{3}}$$

If $b \neq 0$ we can try to solve the quadratic equation by factoring the quadratic polynomial.

eg Solve $x^2-6x+8=0$.

We can factor $x^2-6x+8=(x-2)(x-4)$.

Thus the equation becomes

$$(x-2)(x-4)=0$$

This means that $x-2=0$ or $x-4=0$

so the solutions are $\boxed{x=2 \text{ and } x=4}$.

eg Solve $(x-3)(x-1)=-1$.

First we must expand the lefthand side:

$$\begin{aligned}(x-3)(x-1) &= x(x-1)-3(x-1)\\ &= x^2-x-3x+3\\ &= x^2-4x+3\end{aligned}$$

The equation becomes

$$\begin{aligned}x^2-4x+3&=-1\\ x^2-4x+4&=0\\ (x-2)^2&=0\end{aligned}$$

Hence $x-2=0$ so the only solution is $\boxed{x=2}$.