Section 1.5

We can multiply or divide rational expressions exactly like we multiply or divide fractions.

eg Divide $\dfrac{x^3+27}{x^2-4}$ by $\dfrac{x^2-3x+9}{12-6x}$.

Dividing by fraction is the same as multiplying by its reciprocal, so dividing by a rational expression works the same way. Here we have

$$\frac{x^3+27}{x^2-4}\cdot\frac{12-6x}{x^2-3x+9}$$

$$=\frac{(x+3)(x^2-3x+9)}{(x-2)(x+2)}\cdot\frac{-6(x-2)}{x^2-3x+9}$$

$$\boxed{=\frac{-6(x+3)}{x+2}}$$

To add or subtract rational expressions they must be written over a common denominator. It is usually most efficient to use the <u>lowest</u> common denominator, which can be obtained by factoring.

eg $\frac{4}{x^2-1} - \frac{2}{x^2+x} = \frac{4}{(x-1)(x+1)} - \frac{2}{x(x+1)}$

$$= \frac{4}{(x-1)(x+1)} \cdot \frac{x}{x} - \frac{2}{x(x+1)} \cdot \frac{x-1}{x-1}$$

$$= \frac{4x}{x(x-1)(x+1)} - \frac{2(x-1)}{x(x-1)(x+1)}$$

$$= \frac{4x-2(x-1)}{x(x-1)(x+1)}$$

$$= \frac{4x-(2x-2)}{x(x-1)(x+1)}$$

$$= \frac{4x-2x+2}{x(x-1)(x+1)}$$

$$= \frac{2x+2}{x(x-1)(x+1)}$$

$$= \frac{2(x+1)}{x(x-1)(x+1)}$$

$$\boxed{= \frac{2}{x(x-1)}}$$

We can also write rational expressions in terms of negative exponents.

eg $(x-3)(x+1)^{-3} + x(x+1)^{-2}$

We can rewrite this as

$$\frac{x-3}{(x+1)^3} + \frac{x}{(x+1)^2} = \frac{x-3}{(x+1)^3} + \frac{x}{(x+1)^2}\cdot\frac{x+1}{x+1}$$

$$= \frac{x-3}{(x+1)^3} + \frac{x(x+1)}{(x+1)^3}$$

$$= \frac{(x-3)+(x^2+x)}{(x+1)^3}$$

$$= \frac{x^2+2x-3}{(x+1)^3}$$

$$\boxed{= \frac{(x-1)(x+3)}{(x+1)^3}}$$

A <u>complex</u> rational expression is an expression that includes a rational expression in its numerator, its denominator, or both.

eg $\dfrac{1+1/x}{1-1/x}$ is a complex rational expression

Here we can write $1+\frac{1}{x} = \frac{x}{x}+\frac{1}{x} = \frac{x+1}{x}$

and $1-\frac{1}{x} = \frac{x}{x}-\frac{1}{x} = \frac{x-1}{x}$

Now we can write

$$\frac{1+1/x}{1-1/x} = \frac{\left(\frac{x+1}{x}\right)}{\left(\frac{x-1}{x}\right)} = \frac{x+1}{x} \cdot \frac{x}{x-1}$$

$$\boxed{= \frac{x+1}{x-1}}$$

eg $\dfrac{2}{1+(x+3)^{-1}}$

A different approach is to remove the internal rational expression by the multiplying the numerator and the denominator of the entire expression by the denominator of the internal rational expression.

Here we write

$$\frac{2}{1+(x+3)^{-1}} = \frac{2}{1+1/(x+3)} \cdot \frac{x+3}{x+3}$$

$$= \frac{2(x+3)}{(x+3)+\frac{1}{x+3}\cdot(x+3)}$$

$$= \frac{2(x+3)}{x+3+1}$$

$$\boxed{= \frac{2(x+3)}{x+4}}$$

Section 1.6: Radicals

Def'n: If b is a real number and n is a natural number then an <u>nth root</u> of b is a real number c for which $c^n = b$.

Here, n is the <u>index</u>.

When $n=2$ we have a <u>square root</u>.

When $n=3$ we have a <u>cube root</u>.

Many numbers have a unique nth root which may be positive or negative.

eg Only $0^2 = 0$ so 0 is the only square root of 0.

eg Only $(-4)^3 = -64$ so -4 is the only cube root of -64.

However, some numbers have no nth roots. This occurs when b is negative and n is even.

eg There is no square root of -64.

Furthermore, some numbers have two nth roots, one positive and one negative. This occurs when b is positive and n is even.

eg Both 8 and -8 are square roots of 64 because $8^2 = 64$ and $(-8)^2 = 64$.

Def'n: The principal nth root of a number b is its nth root if n is odd or $b=0$. It is the positive nth root if n is even and $b>0$. We denote the principal nth root as $\sqrt[n]{b}$. We call this a radical.

If $n=2$, we usually write $\sqrt{b}$ rather than $\sqrt[2]{b}$.

eg $\sqrt{64} = 8$

$\sqrt{0} = 0$

$\sqrt[3]{-64} = -4$

$\sqrt{-64}$ is undefined