Section 1.3

eg Factor $3x^4 - 2x^2 - 1$.

Observe that this polynomial has the same form as $Ax^2 + Bx + C$ with x replaced by x^2.
Thus the same strategies should apply, resulting in a factored form $(kx^2+p)(\ell x^2+q)$.

The only integers whose product is -1 are $-1 \cdot 1 = -1$.

Hence we know that $3x^4 - 2x^2 - 1 = (kx^2-1)(\ell x^2+1)$.

The only integers that are positive and have a product of 3 are $1 \cdot 3 = 3$.

Thus either
$$3x^4 - 2x^2 - 1 = (x^2-1)(3x^2+1) = 3x^4 - 2x^2 - 1$$
or
$$3x^4 - 2x^2 - 1 = (3x^2-1)(x^2+1) = 3x^4 + 2x^2 - 1$$

Now we know that
$$3x^4 - 2x^2 - 1 = (x^2-1)(3x^2+1)$$
$$\boxed{= (x-1)(x+1)(3x^2+1)}$$

We often combine strategies for factoring in order to completely factor a polynomial.

eg Factor $7x^3+28x^2+21x$.

First we have $7x^3+28x^2+21x = 7x(x^2+4x+3)$

$$\boxed{= 7x(x+1)(x+3).}$$

A similar approach is <u>factoring by grouping</u>, where we extract common expressions from groups of terms.

eg Factor $x^3-3x^2-4x+12$.

We can group the terms of this polynomial as

$$(x^3-3x^2)+(-4x+12)$$

$$= x^2(x-3)-4(x-3)$$

$$= (x-3)(x^2-4)$$

$$\boxed{= (x-3)(x-2)(x+2).}$$

Section 1.4 : Absolute Value

Def'n : The absolute value (or modulus) of a number x is

$$|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0. \end{cases}$$

eg $\left|\frac{1}{2}\right| = \frac{1}{2}$ because $\frac{1}{2} \geq 0$

$|0| = 0$ because $0 \geq 0$

$|-4| = -(-4) = 4$ because $-4 < 0$

Properties of the absolute value

① For any real number x, $|x| \geq 0$

② $|-x| = |x|$

③ $|xy| = |x| \cdot |y|$

④ $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ for $y \neq 0$

The absolute value bars are treated as brackets for mathematical operations.

eg $|4-6| = |-2| = 2$

$4 - |-6| = 4 - 6 = -2$

If we are given information about any variables present, we can often simplify expressions involving absolute values.

eg Suppose $x < 0$. Simplify $7-|3x|$.

We have $7-|3x| = 7-(|3|\cdot|x|)$

$$= 7-3|x|$$

$$= 7-3\cdot(-x)$$

$$\boxed{= 7+3x}$$

eg Suppose $x \geq 4$. Simplify $|x-4| + |4-x|$.

We have

$$|x-4| = \begin{cases} x-4, & \text{for } x-4 \geq 0 \text{ so } x \geq 4 \\ -(x-4), & \text{for } x-4 < 0 \text{ so } x < 4 \end{cases}$$

and

$$|4-x| = \begin{cases} 4-x, & \text{for } 4-x \geq 0 \text{ so } x \leq 4 \\ -(4-x), & \text{for } 4-x < 0 \text{ so } x > 4 \end{cases}$$

Therefore $|x-4| + |4-x|$

$$= (x-4) + [-(4-x)]$$

$$= x-4-4+x$$

$$\boxed{= 2x-8}.$$

Section 1.5: Rational Expressions

<u>Def'n</u>: A <u>rational expression</u> has the form $\frac{P}{Q}$ where P and Q are both polynomials.

eg $\frac{1}{x^2}$ and $\frac{x^2-1}{x+1}$ are rational expressions

Rational expressions can often be simplified by factoring the polynomials in the numerator and the denominator.

eg Simplify $\frac{x^2-1}{x+1}$.

$$\frac{x^2-1}{x+1} = \frac{(x-1)(x+1)}{x+1} = x-1$$

However, we cannot cancel factors when they are equal to 0, which is true in this case when $x=-1$.

Thus $\frac{x^2-1}{x+1} = x-1$ for $x \neq -1$.