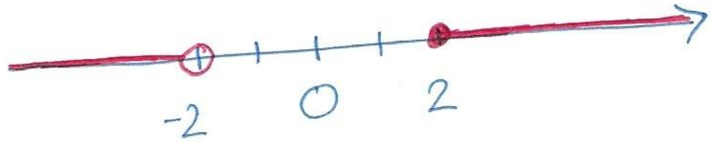
Section 1.1

To denote the combination of two or more non-overlapping intervals, we use the <u>union</u> symbol $\cup$.

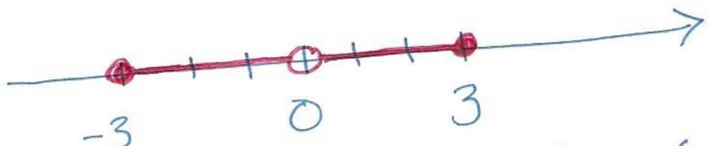
eg The set $A = \{x \in \mathbb{R} \mid x < -2 \text{ or } x \geq 2\}$ can be depicted as



In interval notation, we write this as

$$(-\infty, -2) \cup [2, \infty).$$

eg The set $B = \{x \mid -3 \leq x \leq 3,\ x \neq 0\}$ can be depicted as



In interval notation, this is $[-3, 0) \cup (0, 3]$.

Here we assume that the set B consists of all real numbers that satisfy the given requirements.

Section 1.2: Exponents

Consider an expression of the form b^p. The real number b is the <u>base</u> and the real number p is the <u>exponent</u> (or the <u>power</u>).

If p is a natural number, we define

$$b^p = \underbrace{b \cdot b \cdot b \cdot b \cdots b}_{p \text{ factors}}$$

eg $3^1 = 3$ In fact, $b^1 = b$ for any b

$3^2 = 3 \cdot 3 = 9$ ("squared")

$3^3 = 3 \cdot 3 \cdot 3 = 27$ ("cubed")

$3^4 = 3 \cdot 3 \cdot 3 \cdot 3 = 81$

$(-\frac{1}{2})^3 = (-\frac{1}{2}) \cdot (-\frac{1}{2}) \cdot (-\frac{1}{2}) = -\frac{1}{8}$

$0^2 = 0 \cdot 0 = 0$

Furthermore, $b^0 = 1$ for $b \neq 0$. However, 0^0 is undefined.

eg $3^0 = 1$ $(-\frac{1}{2})^0 = 1$

We also define $b^{-p} = \frac{1}{b^p}$ for $b \neq 0$.

eg $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

$(-\frac{1}{2})^{-3} = \frac{1}{(-\frac{1}{2})^3} = \frac{1}{-1/8} = -8$

Properties of exponents

① $b^p \cdot b^q = b^{p+q}$

eg $(-2)^4 \cdot (-2)^5 = (-2)^{4+5} = (-2)^9 = -512$

② $\dfrac{b^p}{b^q} = b^{p-q}$

eg $\dfrac{3^4}{3^7} = 3^{4-7} = 3^{-3} = \dfrac{1}{3^3} = \dfrac{1}{27}$

eg $\dfrac{x^9}{x^5} = x^{9-5} = x^4$

③ $(bc)^p = b^p c^p$

eg $(5x)^2 = 5^2 x^2 = 25x^2$

Likewise, we can write $(bcd)^p = b^p c^p d^p$, and so on.

④ $(b^p)^q = b^{pq}$

eg $(3^2)^4 = 3^{2\cdot 4} = 3^8 = 6561$

$(x^3)^{-2} = x^{3\cdot(-2)} = x^{-6} = \dfrac{1}{x^6}$

(5) $\left(\frac{b}{c}\right)^p = \frac{b^p}{c^p}$

eg $\left(\frac{x^2}{y^3}\right)^4 = \frac{(x^2)^4}{(y^3)^4} = \frac{x^{2\cdot 4}}{y^{3\cdot 4}} = \frac{x^8}{y^{12}}$

We can use Property (5) to observe that

$$\left(\frac{b}{c}\right)^{-1} = \frac{b^{-1}}{c^{-1}} = \frac{1/b}{1/c} = \frac{1}{b}\cdot\frac{c}{1} = \frac{c}{b}$$

Thus a fraction raised to a power of -1 is equivalent to taking the reciprocal of that fraction.

More generally, $\left(\frac{b}{c}\right)^{-p} = \left(\frac{c}{b}\right)^p = \frac{c^p}{b^p}$.

eg
$$\left(\frac{3x^4}{y}\right)^{-2} = \left(\frac{y}{3x^4}\right)^2$$
$$= \frac{y^2}{(3x^4)^2}$$
$$= \frac{y^2}{3^2(x^4)^2}$$
$$= \frac{y^2}{9x^{4\cdot 2}}$$
$$= \frac{y^2}{9x^8}$$

We want to look for combinations of the properties of exponents to help us simplify exponential expressions.

eg $$\left(\frac{(2x^3)^4}{8x^{-1}y^{-9}}\right)^2 = \left(\frac{2^4(x^3)^4}{8x^{-1}y^{-9}}\right)^2$$
$$= \left(\frac{16\,x^{3\cdot 4}}{8x^{-1}y^{-9}}\right)^2$$
$$= \left(\frac{16x^{12}}{8x^{-1}y^{-9}}\right)^2$$
$$= \left(\frac{2x^{12}}{x^{-1}y^{-9}}\right)^2$$
$$= \left(\frac{2x^{12-(-1)}}{y^{-9}}\right)^2$$
$$= \left(2x^{13}y^{9}\right)^2$$
$$= 2^2x^{13\cdot 2}y^{9\cdot 2}$$
$$= 4x^{26}y^{18}$$

While this kind of simplification can be achieved for expressions involving products and quotients we must be careful when addition or subtraction is present.

eg $$\frac{1}{x^{-1}+y^{-1}} \neq \frac{1}{x^{-1}} + \frac{1}{y^{-1}} = (x^{-1})^{-1} + (y^{-1})^{-1}$$
$$= x + y$$

In fact, $$\frac{1}{x^{-1}+y^{-1}} = \frac{1}{1/x + 1/y}$$
$$= \frac{1}{y/xy + x/xy}$$
$$= \frac{1}{(x+y)/xy} = \frac{xy}{x+y}$$