Section 2.5

eg Find the domain of $f(x) = \sqrt{4-4x} - 6$.

We must have $4-4x \geq 0$

$$4 \geq 4x$$

$$1 \geq x$$

So the domain of $f(x)$ is $D = (-\infty, 1]$.

To sketch the graph of a square root function, we need to find the domain, the vertex, and any x- or y-intercepts. A square root function may have zero or one x-intercepts, and zero or one y-intercepts.

We cannot rely on symmetry to determine additional points on the curve so, if there are no intercepts we identify an additional point on the domain of the function.

eg Sketch the graph of $f(x) = \sqrt{4-4x} - 6$.

We have already found that the domain is $(-\infty, 1]$.

We can rewrite the function as

$$f(x) = \sqrt{4(1-x)} - 6$$

$$= \sqrt{4} \cdot \sqrt{1-x} - 6$$

$$= 2\sqrt{1-x} - 6$$

Thus $a=2$, $h=1$, $k=-6$. The vertex is $(1,-6)$.

To find the y-intercept, we evaluate

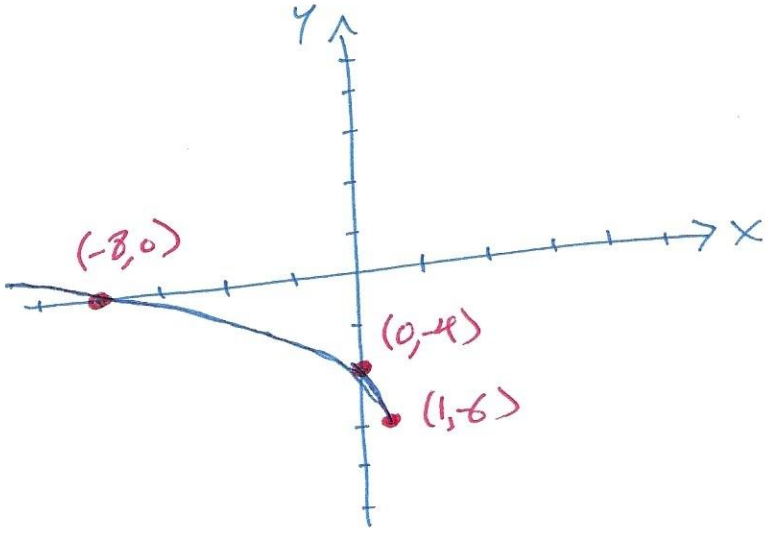
$$f(0) = 2\sqrt{1-0} - 6 = 2 - 6 = -4$$

so the y-intercept is $(0,-4)$.

To find the x-intercept, we set $f(x) = 0$ so

$$2\sqrt{1-x} - 6 = 0$$
$$2\sqrt{1-x} = 6$$
$$\sqrt{1-x} = 3$$
$$(\sqrt{1-x})^2 = 3^2$$
$$1-x = 9$$
$$1-9 = x$$
$$-8 = x$$

So the x-intercept is $(-8,0)$.



eg Sketch the graph of $f(x) = -\sqrt{x+4}$.

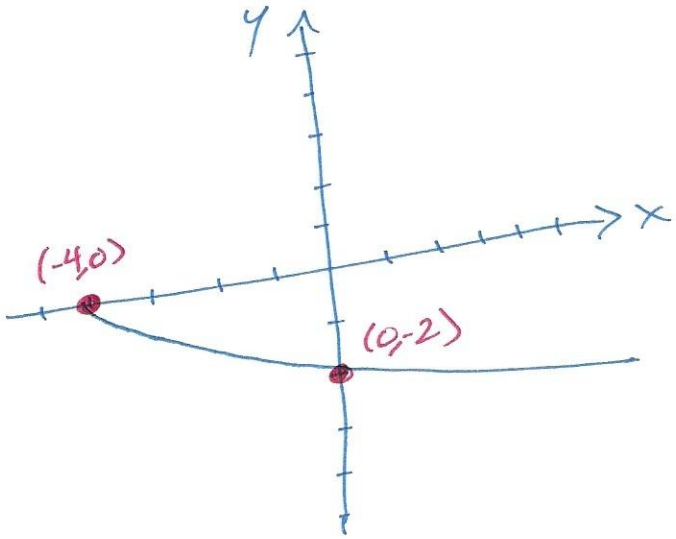
Since this can be written $f(x) = -\sqrt{x-(-4)} + 0$ we know that $a=-1$, $h=-4$, $k=0$. The vertex is $(-4,0)$ which is therefore the only x-intercept.

The domain is found by setting $x+4 \geq 0$ so $x \geq -4$ which means that the domain is $D = [-4, \infty)$.

To find the y-intercept, we evaluate

$$f(0) = -\sqrt{0+4} = -\sqrt{4} = -2$$

so this is the point $(0,-2)$.



eg Sketch the graph of $f(x) = 3\sqrt{x-1} + 2$.

We have $a=3$, $h=1$, $k=2$ so the vertex is the point $(1, 2)$. We must have $x-1 \geq 0$ so $x \geq 1$ and thus the domain is $D = [1, \infty)$.

There is no y-intercept, because $x=0$ is excluded from D. We can also see this since

$$f(0) = 3\sqrt{0-1} + 2 = 3\sqrt{-1} + 2$$

which is undefined.

To find the x-intercept, we set $f(x)=0$ so

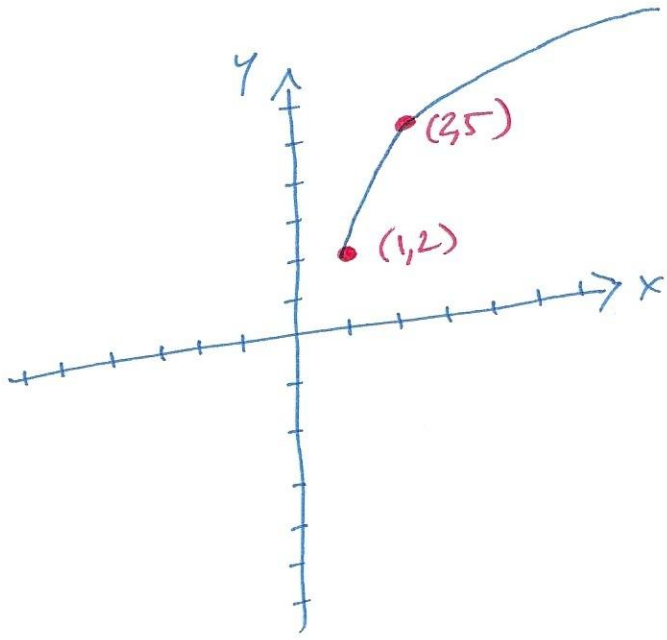
$$3\sqrt{x-1} + 2 = 0$$
$$3\sqrt{x-1} = -2$$
$$\sqrt{x-1} = -\frac{2}{3}$$

But this has no solutions, because $\sqrt{x-1}$ cannot be negative for any x. Hence there is no x-intercept.

We need an additional point, so we choose $x=2$ and get

$$f(2) = 3\sqrt{2-1} + 2$$
$$= 3\sqrt{1} + 2$$
$$= 5$$

so this is the point $(2, 5)$.



Section 2.6: Composite Functions

Functions can be added, subtracted, multiplied or divided by performing the appropriate operation on the corresponding expressions.

eg For $f(x) = 2x^2 + 3x - 1$ and $g(x) = 2 - x$ then

$$f(x) + g(x) = (2x^2 + 3x - 1) + (2 - x)$$
$$= 2x^2 + 2x + 1$$

$$f(x) \cdot g(x) = (2x^2 + 3x - 1) \cdot (2 - x)$$
$$= 4x^2 - 2x^3 + 6x - 3x^2 - 2 + x$$
$$= -2x^3 + x^2 + 7x - 2$$

Another operation available to functions is <u>composition</u>. This is what we obtain when we substitute one function into another function.

eg For $f(x) = 2x^2 + 3x - 1$ and $g(x) = 2 - x$, we can form the composition

$$f(g(x)) = f(2 - x)$$
$$= 2(2-x)^2 + 3(2-x) - 1$$
$$= 2(4 - 4x + x^2) + (6 - 3x) - 1$$
$$= 8 - 8x + 2x^2 + 6 - 3x - 1$$
$$\boxed{= 2x^2 - 11x + 13}$$

We call this a <u>composite function</u>, and denote it by $(fg)(x) = f(g(x))$.