Section 2.5

A parabola always has exactly <u>one</u> y-intercept.

A parabola may have zero, one or two x-intercepts.

eg Find the x- and y-intercepts of the parabola $y = x^2 + 2x - 3$.

To find the y-intercept, we set $x = 0$ so

$$y = 0^2 + 2 \cdot 0 - 3 = -3$$

The y-intercept is the point $(0, -3)$.

To find any x-intercepts, we set

$$x^2 + 2x - 3 = 0$$

$$(x+3)(x-1) = 0$$

$$x + 3 = 0 \quad \text{or} \quad x - 1 = 0$$

$$x = -3 \qquad x = 1$$

Both $(-3, 0)$ and $(1, 0)$ are x-intercepts.

To graph a parabola, we find the vertex, the y-intercept and any x-intercepts. In addition, we identify a point on the opposite side of the vertex from the y-intercept using the symmetry of the parabola.

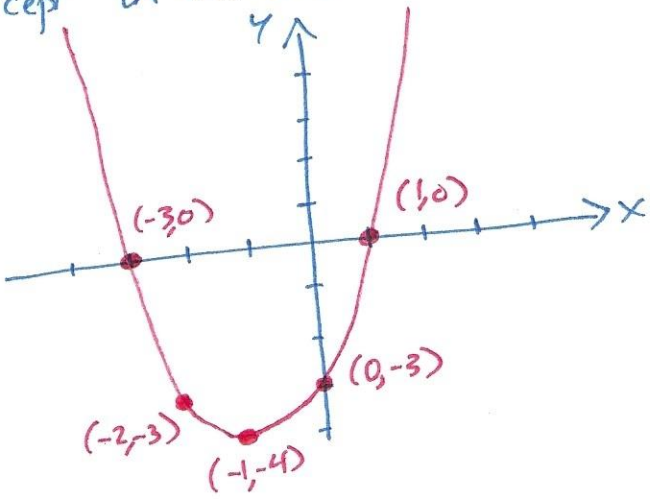
eg Sketch the graph of $f(x) = x^2 + 2x - 3$.

We know that the vertex is $(-1, -4)$

the y-intercept is $(0, -3)$

the x-intercepts are $(-3, 0)$ and $(1, 0)$

A point opposite the y-intercept on the other side of the vertex is $(-2, -3)$.



If the vertex is the y-intercept and there are no x-intercepts, we choose any non-zero value of x to find a point on the parabola, and use the symmetry of the parabola to identify the point that lies on the opposite side of the vertex from it.

eg Sketch the graph of the parabola $y = -x^2 - 2$.

Here $a = -1$, $b = 0$, $c = -2$ so

$$h = -\frac{b}{2a} = -\frac{0}{2 \cdot (-1)} = 0$$

Thus $k = -0^2 - 2 = -2$.

So the vertex is the point $(0, -2)$ which is therefore also the y-intercept.

To find any x-intercepts we set

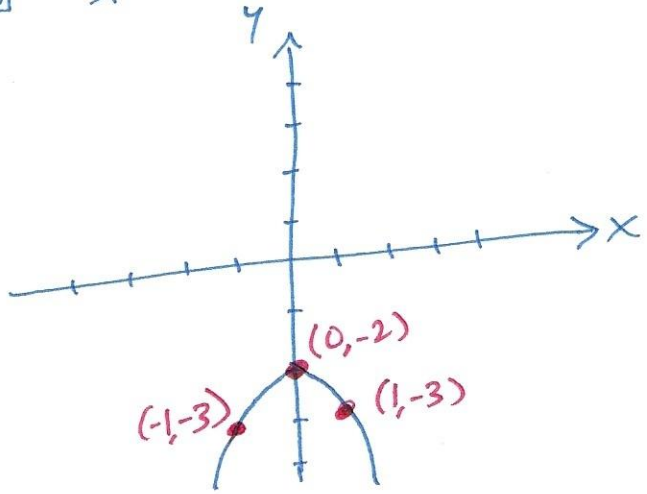
$$-x^2 - 2 = 0$$

$$-2 = x^2$$

which has no solutions. Hence there are <u>no</u> x-intercepts.
We need an additional point, so we choose $x = 1$:

$$y = -1^2 - 2 = -3$$

So this is the point $(1, -3)$. Then $(-1, -3)$ is also a point on the graph since it lies opposite $(1, -3)$ on the other side of the vertex.



Observe that the parabola $y = ax^2 + bx + c$ will always open upward if $a > 0$, and will always open downward if $a < 0$.

eg Sketch the graph of $f(x) = 4x - 4x^2 - 1$.

We can rewrite this function as

$$f(x) = -4x^2 + 4x - 1.$$

Then $a = -4$, $b = 4$, $c = -1$.

Since $a < 0$, the parabola opens downward.

Next, $h = -\frac{b}{2a} = -\frac{4}{2 \cdot (-4)} = \frac{1}{2}$

$$k = f\left(\tfrac{1}{2}\right) = 4 \cdot \tfrac{1}{2} - 4 \cdot \left(\tfrac{1}{2}\right)^2 - 1$$

$$= 2 - 1 - 1$$

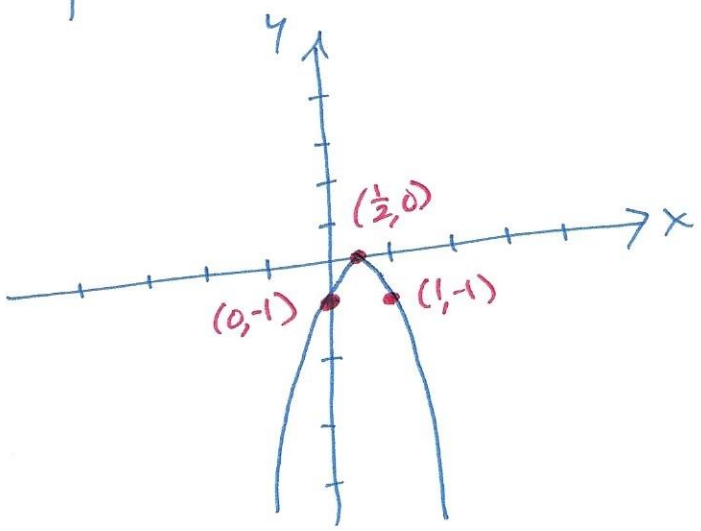
$$= 0$$

So the vertex is the point $(\frac{1}{2}, 0)$. This is therefore the only x-intercept.

Now we find the y-intercept by setting $x = 0$, so

$$f(0) = 4 \cdot 0 - 4 \cdot 0^2 - 1 = -1$$

which is the point $(0, -1)$. The point $(1, -1)$ also lies on the parabola from symmetry.



Defn: A linear absolute value function is any function that can be written in the form

$$f(x) = a|x-h| + k$$

for $a \neq 0$.

First consider $f(x) = |x|$. This is really a piecewise function because it can be written

$$f(x) = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}.$$

To graph a piecewise function, we sketch the graph of each of its definitions, but keep only the part of each graph that corresponds to the appropriate values of x.