Section 1.1: Numbers

Def'n: A set is a collection of elements (for our purposes, the elements will always be numbers).

One way to denote a set is to list its elements in between braces, separated by commas.

eg The set of even numbers between 5 and 13 is

$$A = \{6, 8, 10, 12\}.$$

eg The set of even numbers between 5 and 93 is

$$B = \{6, 8, 10, 12, \ldots, 92\}.$$

eg The set of all even numbers greater than 5 is

$$C = \{6, 8, 10, 12, \ldots\}$$

If x is an element of a set S, we write $x \in S$.

If x is not an element of S, we write $x \notin S$.

eg In the set C above, $6 \in C$ but $4 \notin C$.

Alternatively, we can define a set by establishing a formula or other description for its elements.

eg For set C above, we could also write

$$C = \{2n \mid n \geq 3\}.$$

Def'n: The set of natural numbers is

$$\mathbb{N} = \{1, 2, 3, 4, \dots\}$$

Def'n: The set of integers is

$$\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3 \dots\}$$

Def'n: The set $\mathbb{Q}$ of rational numbers is the set of all numbers that can be written as a ratio $\frac{p}{q}$ where p is an integer and q is a non-zero integer.

eg Rational numbers include

$$\frac{3}{1} = 3, \quad \frac{-7}{1} = -7, \quad \frac{0}{1} = 0$$

$$\frac{1}{2} = 0.5, \quad \frac{4}{9} = 0.444\ldots = 0.\overline{4}$$

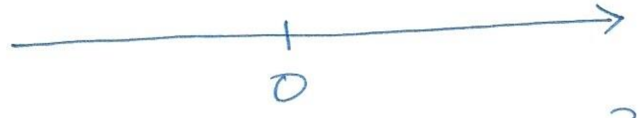
Any number with a terminating or infinitely repeating decimal representation is a rational number.

eg $\sqrt{2} = 1.414213562\ldots$ does not have a terminating or infinitely repeating decimal representation, so it is not a rational number.

A number with a decimal representation that neither terminates nor infinitely repeats is an irrational number.

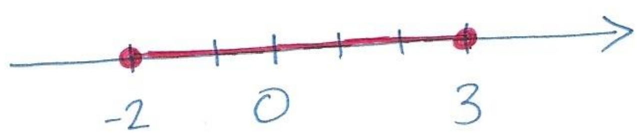
Def'n: The set $\mathbb{R}$ of real numbers consists of all rational numbers and all irrational numbers.

We can illustrate the real numbers using a number line, which is a line with a direction and a chosen reference point for 0.

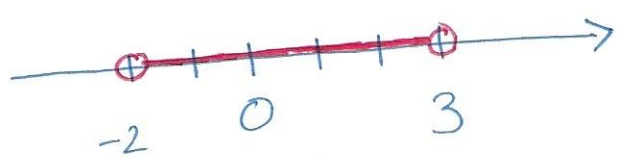


eg The set $A = \{x \in \mathbb{R} \mid -2 \leq x \leq 3\}$

This set can be represented on the number line as



eg The set $B = \{x \in \mathbb{R} \mid -2 < x < 3\}$ can be represented as



A contiguous set of real numbers is called an interval.

We can more compactly denote an interval as follows:

1. A closed interval: $\{x \in \mathbb{R} \mid a \leq x \leq b\} = [a, b]$
2. An open interval: $\{x \in \mathbb{R} \mid a < x < b\} = (a, b)$

③ A <u>half-open</u> interval : $\{x \in \mathbb{R} \mid a \leq x < b\} = [a, b)$

④ A <u>half-open</u> interval : $\{x \in \mathbb{R} \mid a < x \leq b\} = (a, b]$

eg $A = \{x \in \mathbb{R} \mid -2 \leq x \leq 3\} = [-2, 3]$

$B = \{x \in \mathbb{R} \mid -2 < x < 3\} = (-2, 3)$

To indicate that there is no right endpoint to an interval, we use the symbol ∞. To indicate that there is no left endpoint, we use $-\infty$.

eg The set $C = \{x \in \mathbb{R} \mid x \geq 0\} = [0, \infty)$

It can be represented by

Thus we can also write $\mathbb{R} = (-\infty, \infty)$.