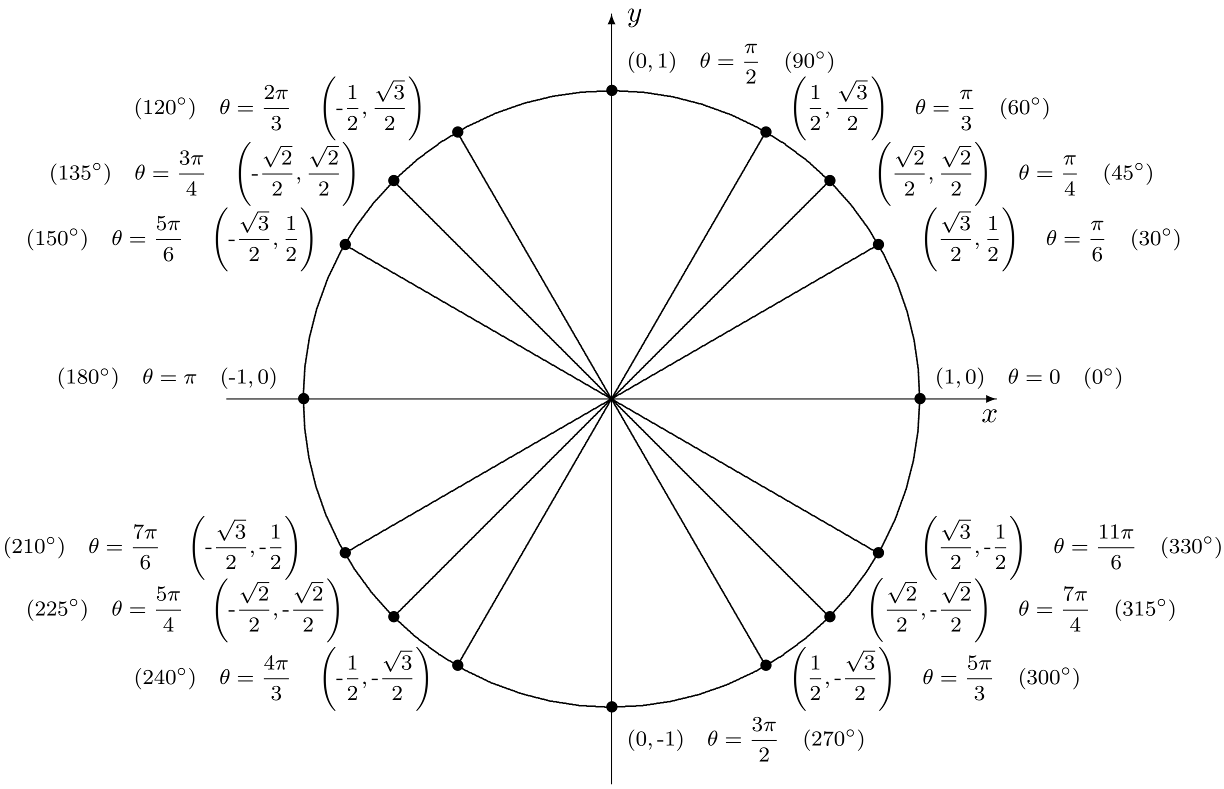
Basics of Trigonometry

One way to think about the trigonometric functions is in terms of the unit circle (that is, the circle centred at the origin, of radius 1). If (x, y) is a point on the unit circle corresponding to an angle θ, then

$$x = \cos(\theta) \quad \text{and} \quad y = \sin(\theta).$$

Note that we begin measuring θ from the point $(1, 0)$ (so this corresponds to the angle $\theta = 0$). If $\theta > 0$ then it is measured counterclockwise, and if $\theta < 0$ then it is measured clockwise.



Furthermore, observe that both sine and cosine are periodic with period 2π, so for any angle θ, we have

$$\cos(\theta + 2\pi) = \cos(\theta) \quad \text{and} \quad \sin(\theta + 2\pi) = \sin(\theta).$$

Additionally, from the graphs of the cosine and sine functions, we can see that

$$\cos(-\theta) = \cos(t) \quad \text{and} \quad \sin(-\theta) = -\sin(\theta).$$

There are a number of useful relationships between the trigonometric functions (called identities). On the next page is a list of the most important.

Basic Identities

- $\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$
- $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)} = \dfrac{1}{\tan(\theta)}$
- $\sec(\theta) = \dfrac{1}{\cos(\theta)}$
- $\csc(\theta) = \dfrac{1}{\sin(\theta)}$
- $\tan(-\theta) = -\tan(\theta)$
- $\sin^2(\theta) + \cos^2(\theta) = 1$
- $\tan^2(\theta) + 1 = \sec^2(\theta)$
- $1 + \cot^2(\theta) = \csc^2(\theta)$

Sum and Difference Identities

- $\cos(\alpha + \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta)$
- $\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) + \sin(\alpha)\sin(\beta)$
- $\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta)$
- $\sin(\alpha - \beta) = \sin(\alpha)\cos(\beta) - \cos(\alpha)\sin(\beta)$

Double and Half-Angle Identities

- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$
- $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + \cos(\theta)}{2}}$
- $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos(\theta)}{2}}$