SOLUTIONS

1. (a) We rewrite the equation of the circle in standard form:

$$x^2 + y^2 + 10x - 4y - 7 = 0$$
$$(x^2 + 10x + 25) + (y^2 - 4y + 4) - 7 - 25 - 4 = 0$$
$$(x + 5)^2 + (y - 2)^2 = 36.$$

This means that the centre of the circle is $(-5, 2)$ and its radius is $\sqrt{36} = 6$.

(b) We rewrite the equation of the circle in standard form. Note that, in this case, we need to complete the square in x but not in y. We have

$$x^2 + y^2 - 2x - 5 = 0$$
$$(x^2 - 2x + 1) + y^2 - 5 - 1 = 0$$
$$(x - 1)^2 + y^2 = 6.$$

This means that the centre of the circle is $(1, 0)$ and its radius is $\sqrt{6}$.

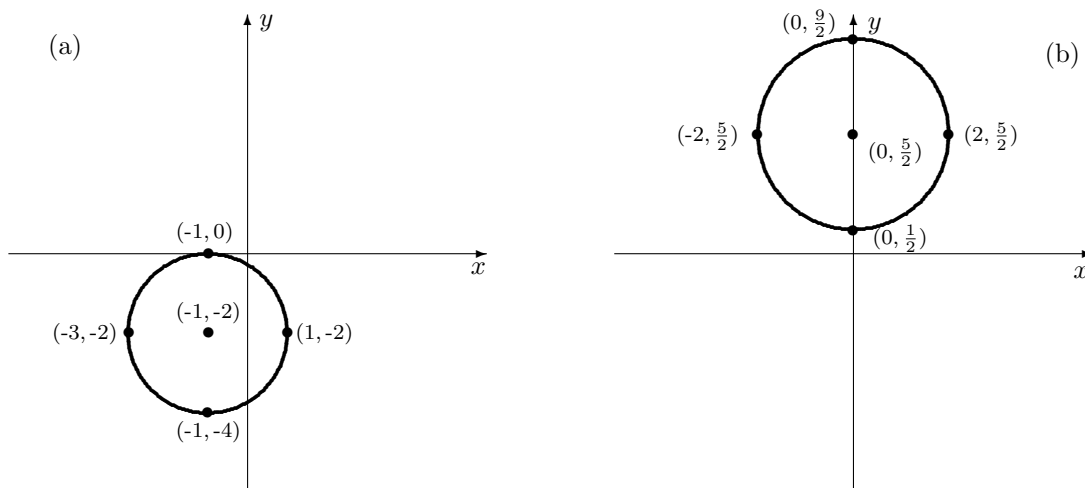(c) We rewrite the equation of the circle in standard form by first dividing each side by 9, giving

$$9x^2 + 9y^2 - 6x + 12y - 139 = 0$$
$$\left(x^2 - \frac{2}{3}x\right) + \left(y^2 + \frac{4}{3}y\right) - \frac{139}{9} = 0$$
$$\left(x^2 - \frac{2}{3}x + \frac{1}{9}\right) + \left(y^2 + \frac{4}{3}y + \frac{4}{9}\right) - \frac{139}{9} - \frac{1}{9} - \frac{4}{9} = 0$$
$$\left(x - \frac{1}{3}\right)^2 + \left(y + \frac{2}{3}\right)^2 = 16.$$

Now we can see that the centre of the circle is $\left(\frac{1}{3}, -\frac{2}{3}\right)$ and its radius is $\sqrt{16} = 4$.

2. (a) We rewrite the circle in standard form:

$$x^2 + y^2 + 2x + 4y + 1 = 0$$
$$(x^2 + 2x + 1) + (y^2 + 4y + 4) + 1 - 1 - 4 = 0$$
$$(x + 1)^2 + (y + 2)^2 = 4.$$

This circle has centre $(-1, -2)$ and radius $\sqrt{4} = 2$. Thus four points on the circle are $(1, -2)$, $(-3, -2)$, $(-1, 0)$ and $(-1, -4)$. We plot these points and sketch the graph, shown below.

(a)

(b)

(b) First, divide both sides by 4 to rewrite the equation of the circle as

$$x^2 + y^2 - 5y + \frac{9}{4} = 0.$$

There is no x-term, so we do not need to complete the square in x. We do need to complete the square in y, so we have

$$x^2 + (y^2 - 5y) + \frac{9}{4} = 0$$

$$x^2 + \left(y^2 - 5y + \frac{25}{4}\right) - \frac{25}{4} + \frac{9}{4} = 0$$

$$x^2 + \left(y - \frac{5}{2}\right)^2 = 4.$$

This circle has centre $(0, \frac{5}{2})$ and radius $\sqrt{4} = 2$. This means that four points on the circle are $(0, \frac{9}{2})$, $(0, \frac{1}{2})$, $(2, \frac{5}{2})$ and $(-2, \frac{5}{2})$. Now can sketch the circle, shown above.