MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 5.3 **Math 1090** FALL 2009

SOLUTIONS

1. (a) First we find the critical numbers. The corresponding equation is

$$\begin{aligned} x^2 &= 3 - 2x \\ x^2 + 2x - 3 &= 0 \\ (x+3)(x-1) &= 0 \end{aligned}$$

so the critical numbers are $x = -3$ and $x = 1$; note that because the original inequality is strict, neither of these are included in the solution.

We now check to see if the original inequality is satisfied on each of the resulting intervals. On $(-\infty, -3)$ we try $x = -4$ and see that $x^2 = 16$ while $3 - 2x = 11$, so the inequality fails. On $(-3, 1)$ we try $x = 0$ so $x^2 = 0$ and $3 - 2x = 3$, so the inequality is satisfied and this is part of the solution. On $(1, \infty)$ we try $x = 2$ and see that $x^2 = 4$ and $3 - 2x = -1$, so again the inequality fails. Hence the solution is $(-3, 1)$.

(b) First we must find the critical numbers, by solving the correspond equation

$$\begin{aligned} (x-2)^2 &= 1 \\ x^2 - 4x + 4 &= 1 \\ x^2 - 4x + 3 &= 0 \\ (x-3)(x-1) &= 0. \end{aligned}$$

Hence the critical numbers are $x = 1$ and $x = 3$. The inequality is strict, so these are not part of the solution. Now we need to consider the intervals $(-\infty, 1)$, $(1, 3)$ and $(3, \infty)$.

On $(-\infty, 1)$ try $x = 0$. Then

$$(x-2)^2 = (0-2)^2 = 4 > 1,$$

so this is part of the solution.

On $(1, 3)$, try $x = 2$. Then

$$(x-2)^2 = (2-2)^2 = 0 < 1,$$

so this is not part of the solution.

On $(3, \infty)$, try $x = 4$. Then

$$(x-2)^2 = (4-2)^2 = 4 > 1,$$

so this is also part of the solution.

Hence the solution of the inequality is $(-\infty, 1) \cup (3, \infty)$.

2. (a) Note that $x = 3$ is a critical number because it makes the denominator zero. To find other critical numbers we solve the corresponding equation

$$\begin{aligned} \frac{3}{3-x} &= 5 \\ \frac{3}{3-x} &= \frac{5(3-x)}{3-x} \\ 3 &= 15 - 5x \\ 5x &= 12 \\ x &= \frac{12}{5}. \end{aligned}$$

Note that this is not part of the solution, again because the given inequality is strict. On $\left(-\infty, \frac{12}{5}\right)$ we try $x = 0$ and see that $\frac{3}{3-x} = 1 < 5$, so the inequality is satisfied and this is part of the solution. On $\left(\frac{12}{5}, 3\right)$ we try $x = \frac{13}{5}$ and see that $\frac{3}{3-x} = \frac{15}{2} > 5$ so the inequality fails. On $(3, \infty)$ we try $x = 4$ so $\frac{3}{3-x} = -3 < 5$ so again the inequality is satisfied, and therefore this is also part of the solution. The full solution, then, is $\left(-\infty, \frac{12}{5}\right) \cup (3, \infty)$.

(b) First note that both $x = 4$ and $x = -\frac{3}{2}$ are critical numbers, because they make the denominators zero. To find any other critical numbers, we solve

$$\begin{aligned} \frac{1}{x-4} &= \frac{1}{2x+3} \\ \frac{2x+3}{(x-4)(2x+3)} &= \frac{x-4}{(x-4)(2x+3)} \\ 2x+3 &= x-4 \\ x &= -7. \end{aligned}$$

Note that this is part of the solution, because the inequality is not strict.

This time, then, we have four intervals to check. On $(-\infty, -7)$ we try $x = -8$ so $\frac{1}{x-4} = -\frac{1}{12}$ and $\frac{1}{2x+3} = -\frac{1}{13}$ so the inequality fails. On $\left(-7, -\frac{3}{2}\right)$ we try $x = -2$ so $\frac{1}{x-4} = -\frac{1}{6}$ and $\frac{1}{2x+3} = -1$, which means that the inequality is satisfied and this is part of the solution. On $\left(-\frac{3}{2}, 4\right)$ we try $x = 0$ and get $\frac{1}{x-4} = -\frac{1}{4}$ while $\frac{1}{2x+3} = \frac{1}{3}$ so the inequality fails. Finally, on $(4, \infty)$ we try $x = 5$ so $\frac{1}{x-4} = 1$ and $\frac{1}{2x+3} = \frac{1}{13}$, meaning that the inequality is satisfied and this is also part of the solution. Thus the full solution is $\left[-7, -\frac{3}{2}\right) \cup (4, \infty)$.