MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 5.2 **Math 1090** FALL 2009

SOLUTIONS

1. (a) $\dfrac{3x+9}{x^2-1}\cdot\dfrac{x+1}{2x^2+5x-3}=\dfrac{3(x+3)}{(x-1)(x+1)}\cdot\dfrac{x+1}{(2x-1)(x+3)}=\dfrac{3}{(x-1)(2x-1)}$

   (b) $$\begin{aligned}\frac{2}{2x-5}+\frac{1}{x+1}&=\frac{2(x+1)}{(2x-5)(x+1)}+\frac{1(2x-5)}{(x+1)(2x-5)}=\frac{2x+2+2x-5}{(2x-5)(x+1)}\\&=\frac{4x-3}{(2x-5)(x+1)}\end{aligned}$$

   (c) $$\begin{aligned}\frac{1}{9-x^2}+\frac{1}{x-3}&=\frac{1}{-(x-3)(x+3)}+\frac{1}{x-3}=\frac{-1}{(x-3)(x+3)}+\frac{1(x+3)}{(x-3)(x+3)}\\&=\frac{-1+x+3}{(x-3)(x+3)}=\frac{x+2}{(x-3)(x+3)}\end{aligned}$$

   (d) $$\begin{aligned}\frac{x-2}{x-4}-\frac{x+3}{x^2-2x-8}&=\frac{x-2}{x-4}-\frac{x+3}{(x-4)(x+2)}=\frac{(x-2)(x+2)}{(x-4)(x+2)}-\frac{x+3}{(x-4)(x+2)}\\&=\frac{x^2-4-x-3}{(x-4)(x+2)}=\frac{x^2-x-7}{(x-4)(x+2)}\end{aligned}$$

   (e) $$\frac{\frac{2}{x+2}-1}{1-\frac{3}{x+3}}=\frac{\frac{2}{x+2}-\frac{x+2}{x+2}}{\frac{x+3}{x+3}-\frac{3}{x+3}}=\frac{\frac{2-x-2}{x+2}}{\frac{x+3-3}{x+3}}=\frac{\frac{-x}{x+2}}{\frac{x}{x+3}}=\frac{-x}{x+2}\cdot\frac{x+3}{x}=\frac{-(x+3)}{x+2}$$

   (f) $$\begin{aligned}x(x-4)^{-2}+3(x-4)^{-1}&=\frac{x}{(x-4)^2}+\frac{3}{x-4}=\frac{x}{(x-4)^2}+\frac{3(x-4)}{(x-4)^2}\\&=\frac{x+3x-12}{(x-4)^2}=\frac{4(x-3)}{(x-4)^2}\end{aligned}$$

2. (a) First observe that

$$\begin{aligned}\frac{2}{x-5}+\frac{1}{x+5}&=\frac{5x-5}{x^2-25}\\ \frac{2(x+5)}{(x-5)(x+5)}+\frac{1(x-5)}{(x-5)(x+5)}&=\frac{5x-5}{(x-5)(x+5)}\\ 2x+10+x-5&=5x-5\\ -2x&=-10\\ x&=5.\end{aligned}$$

However, $x=5$ fails upon substitution into the original equation (it results in division by zero) so this equation has no solutions.

(b) This time we have

$$\begin{aligned}\frac{2}{x-5}+\frac{1}{x+5}&=\frac{5x-1}{x^2-25}\\ \frac{2(x+5)}{(x-5)(x+5)}+\frac{1(x-5)}{(x-5)(x+5)}&=\frac{5x-1}{(x-5)(x+5)}\\ 2x+10+x-5&=5x-1\\ -2x&=-6\\ x&=3.\end{aligned}$$

This value of x works upon substitution into the original equation, and so $x = 3$ is the only solution.

(c) We have

$$\begin{aligned}\frac{1}{x}+\frac{1}{x-6}&=-\frac{2}{x^2}\\ \frac{x(x-6)}{x^2(x-6)}+\frac{x^2}{x^2(x-6)}&=-\frac{2(x-6)}{x^2(x-6)}\\ x^2-6x+x^2&=-2x+12\\ 2x^2-4x-12&=0\\ x^2-2x-6&=0.\end{aligned}$$

Since this trinomial is irreducible, we use the quadratic formula and see that

$$x=\frac{2\pm\sqrt{2^2-4(1)(-6)}}{2(1)}=\frac{2\pm\sqrt{28}}{2}=\frac{2\pm2\sqrt{7}}{2}=1\pm\sqrt{7}.$$

Thus the equation has two solutions: $x = 1 + \sqrt{7}$ and $x = 1 - \sqrt{7}$.

3. Any value of x will be in the domain D as long as it does not make the denominator zero. In other words, we must solve the equation

$$\begin{aligned}x^2+4x-12&=0\\ (x+6)(x-2)&=0\end{aligned}$$

so $x = -6$ and $x = 2$. Hence

$$D_f=\{x\,|\,x\neq-6, x\neq 2\}.$$

4. (a) First we have

$$\begin{aligned}(f \circ g)(x) = f(g(x)) = f\left(\frac{1}{x-3}\right) &= \frac{1}{\frac{1}{x-3}+3} \\ &= \frac{1}{\frac{1+3(x-3)}{x-3}} \\ &= \frac{x-3}{3x-8}.\end{aligned}$$

Note that this function is undefined if $x = \frac{8}{3}$, while $g(x)$ is undefined if $x = 3$. Thus

$$D_{f \circ g} = \left\{x \,|\, x \neq \frac{8}{3}, x \neq 3\right\}.$$

(b) The composition is

$$\begin{aligned}(f \circ g)(x) = f(g(x)) &= f(2-x) \\ &= \frac{2-x}{(2-x)^2-4} \\ &= \frac{2-x}{x^2-4x}.\end{aligned}$$

In this case, $g(x)$ is always defined (it's a polynomial) but the composite function will be undefined if $x^2 - 4x = x(x-4) = 0$, so if $x = 0$ or $x = 4$. Thus

$$D_{f \circ g} = \{x \,|\, x \neq 0, x \neq 4\}.$$

(c) This time,

$$\begin{aligned}(f \circ g)(x) = f(g(x)) = f\left(\sqrt{4-x}\right) &= \frac{2}{1-\left(\sqrt{4-x}\right)^2} \\ &= \frac{2}{1-(4-x)} \\ &= \frac{2}{x-3}.\end{aligned}$$

This function is undefined if $x = 3$, while $g(x)$ is only defined if $4 - x \geq 0$ so $x \leq 4$. Hence

$$D_{f \circ g} = \{x \,|\, x \leq 4, x \neq 3\}.$$

5. We simply have

$$
\begin{aligned}
(f \circ f)(x) = f(f(x)) &= f\left(\frac{2}{3x+4}\right) \\
&= \frac{2}{3\left(\frac{2}{3x+4}\right)+4} \\
&= \frac{2}{\frac{6}{3x+4}+4} \\
&= \frac{2}{\frac{6+4(3x+4)}{3x+4}} \\
&= \frac{2}{\frac{12x+22}{3x+4}} \\
&= \frac{2(3x+4)}{12x+22} \\
&= \frac{3x+4}{6x+11}.
\end{aligned}
$$

This function is undefined for $x = -\frac{11}{6}$ while $f(x)$ is undefined for $x = -\frac{4}{3}$. Thus

$$D_{f \circ f} = \left\{ x \,\middle|\, x \neq -\frac{11}{6}, x \neq -\frac{4}{3} \right\}.$$

6. Observe that

$$R_{f^{-1}} = D_f = \{x \,|\, x \geq 3, x \neq 5\}.$$