MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Test 4	**Math 1090**	Fall 2009

SOLUTIONS

[3] 1. (a) The natural exponential function is the function $f(x) = e^x$, so its base is e.

[3] (b) Any exponential function has range $R = (0, \infty)$ and a y-intercept at $(0, 1)$. (Its domain $D = \mathbb{R}$.)

[3] (c) $(-32)^{\frac{3}{5}} = \left(\sqrt[5]{-32}\right)^3 = (-2)^3 = -8$

[3] (d) $(-16)^{\frac{3}{4}}$ is undefined, because we cannot take an even root of a negative number

[3] (e) $27^{-\frac{2}{3}} = \dfrac{1}{27^{\frac{2}{3}}} = \dfrac{1}{\left(\sqrt[3]{27}\right)^2} = \dfrac{1}{3^2} = \dfrac{1}{9}$

[7] 2. We use the double-angle formula

$$\cos(2t) = 1 - 2\sin^2(t),$$

so

$$\begin{aligned}
\cos(2t) - 2\sin^2(t) &= 0 \\
[1 - 2\sin^2(t)] - 2\sin^2(t) &= 0 \\
1 - 4\sin^2(t) &= 0 \\
\sin^2(t) &= \frac{1}{4} \\
\sin(t) &= \pm\sqrt{\frac{1}{4}} = \pm\frac{1}{2}.
\end{aligned}$$

When $\sin(t) = \frac{1}{2}$, $t = \frac{\pi}{6}$ and $t = \frac{5\pi}{6}$. When $\sin(t) = -\frac{1}{2}$, $t = \frac{7\pi}{6}$ and $t = \frac{11\pi}{6}$.

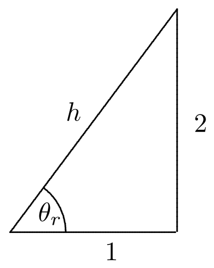
[6] 3. There are several possible ways to write $\frac{13\pi}{12}$ as a sum or difference of special angles, including

$$\begin{aligned}
\frac{13\pi}{12} &= \frac{9\pi}{12} + \frac{4\pi}{12} = \frac{3\pi}{4} + \frac{\pi}{3}, \\
\frac{13\pi}{12} &= \frac{10\pi}{12} + \frac{3\pi}{12} = \frac{5\pi}{6} + \frac{\pi}{4}, \\
\frac{13\pi}{12} &= \frac{16\pi}{12} - \frac{3\pi}{12} = \frac{4\pi}{3} - \frac{\pi}{4}.
\end{aligned}$$

Using the first of these, we now have

$$\begin{aligned}\cos\left(\frac{13\pi}{12}\right) &= \cos\left(\frac{3\pi}{4}+\frac{\pi}{3}\right)\\ &= \cos\left(\frac{3\pi}{4}\right)\cos\left(\frac{\pi}{3}\right) - \sin\left(\frac{3\pi}{4}\right)\sin\left(\frac{\pi}{3}\right)\\ &= \left(-\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2} - \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2}\\ &= -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}\\ &= \frac{-\sqrt{2}-\sqrt{6}}{4}.\end{aligned}$$

[6] 4. Let θ_r be the reference angle for θ, so $\tan(\theta_r) = 2$. We draw a diagram:



By the Pythagorean theorem,

$$\begin{aligned}h^2 &= 2^2 + 1^2 = 5\\ h &= \sqrt{5}.\end{aligned}$$

Thus $\cos(\theta_r) = \frac{1}{\sqrt{5}} = \frac{\sqrt{5}}{5}$. Since θ is in the fourth quadrant, $\cos(\theta) > 0$ and therefore

$$\cos(\theta) = \frac{\sqrt{5}}{5}$$

as well.

Now we have

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos(\theta)}{2}}$$
$$= \pm\sqrt{\frac{1-\frac{\sqrt{5}}{5}}{2}}$$
$$= \pm\sqrt{\frac{\frac{5-\sqrt{5}}{5}}{2}}$$
$$= \pm\sqrt{\frac{5-\sqrt{5}}{10}}.$$

Finally, observe that if θ is in the fourth quadrant then

$$\frac{3\pi}{2} < \theta < 2\pi$$
$$\frac{3\pi}{4} < \frac{\theta}{2} < \pi,$$

which places $\frac{\theta}{2}$ in the second quadrant, where sine is positive. Thus

$$\sin\left(\frac{\theta}{2}\right) = \sqrt{\frac{5-\sqrt{5}}{10}}.$$

[6] 5. We can write all of the factors as powers of 3:

$$9^{3-2x} = 3 \cdot 27^{x-3}$$
$$(3^2)^{3-2x} = 3^1 \cdot (3^3)^{x-3}$$
$$3^{6-4x} = 3^1 \cdot 3^{3x-9}$$
$$3^{6-4x} = 3^{3x-8}$$
$$6-4x = 3x-8$$
$$-7x = -14$$
$$x = 2.$$