# MEMORIAL UNIVERSITY OF NEWFOUNDLAND

## DEPARTMENT OF MATHEMATICS AND STATISTICS

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Section 4.5 | **Math 1090** | Fall 2009

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## SOLUTIONS

1. (a) $\log_5(125) = \log_5(5^3) = 3$

   (b) $\log_3(-81)$ is undefined, because the argument of any logarithm must be positive

   (c) $\log_2\left(\dfrac{1}{16}\right) = \log_2(2^{-4}) = -4$

   (d) $\log_9(3) = \log_9(9^{\frac{1}{2}}) = \frac{1}{2}$

   (e) $\log_4\left(\dfrac{1}{32}\right) = \log_4(4^{-\frac{5}{2}}) = -\dfrac{5}{2}$

   (f) $\log_3(9\sqrt{3}) = \log_3(3^2 \cdot 3^{\frac{1}{2}}) = \log_3(3^{\frac{5}{2}}) = \frac{5}{2}$

2. (a) $2^{5\log_2(3)} = 2^{\log_2(3^5)} = 2^{\log_2(243)} = 243$

   (b) $9^{\log_3(7)} = (3^2)^{\log_3(7)} = 3^{2\log_3(7)} = 3^{\log_3(7^2)} = 3^{\log_3(49)} = 49$

   (c) $\left(\dfrac{1}{2}\right)^{\log_4(5)} = (4^{-\frac{1}{2}})^{\log_4(5)} = 4^{-\frac{1}{2}\log_4(5)} = 4^{\log_4(5^{-\frac{1}{2}})} = 5^{-\frac{1}{2}} = \dfrac{1}{\sqrt{5}} = \dfrac{\sqrt{5}}{5}$

   (d) $e^{3\ln(x)} = e^{\ln(x^3)} = x^3$

3. We have

$$\begin{aligned}
\log_2\left(\frac{x^3 4^y}{2\sqrt{z}}\right) &= \log_2(x^3 4^y) - \log_2(2\sqrt{z}) \\
&= \log_2(x^3) + \log_2(4^y) - [\log_2(2) + \log_2(\sqrt{z})] \\
&= 3\log_2(x) + y\log_2(4) - \log_2(2) - \frac{1}{2}\log_2(z) \\
&= 3\log_2(x) + 2y - 1 - \frac{1}{2}\log_2(z).
\end{aligned}$$

4. We can write

$$\begin{aligned}
\ln(x) + \frac{1}{2}\ln(y) - 5\ln(z) &= \ln(x) + \ln(\sqrt{y}) - \ln(z^5) \\
&= \ln(x\sqrt{y}) - \ln(z^5) \\
&= \ln\left(\frac{x\sqrt{y}}{z^5}\right).
\end{aligned}$$

5. (a) We have

$$\begin{aligned}
\log_4(x+3) &= 2 - \log_4(x-3) \\
\log_4(x+3) + \log_4(x-3) &= 2 \\
\log_4[(x+3)(x-3)] &= 2 \\
\log_4(x^2-9) &= 2 \\
x^2 - 9 &= 4^2 \\
x^2 - 9 &= 16 \\
x^2 &= 25 \\
x &= \pm 5.
\end{aligned}$$

However, $x = -5$ results in negative arguments of the logarithms in the original equation; hence the only solution is $x = 5$.

(b) We have

$$\begin{aligned}
\frac{1}{2}\ln(7-3x) - \ln(1-x) &= 0 \\
\ln\left(\sqrt{7-3x}\right) - \ln(1-x) &= 0 \\
\ln\left(\frac{\sqrt{7-3x}}{1-x}\right) &= 0 \\
\frac{\sqrt{7-3x}}{1-x} &= e^0 \\
\frac{\sqrt{7-3x}}{1-x} &= 1 \\
\sqrt{7-3x} &= 1-x \\
7-3x &= (1-x)^2 \\
7-3x &= x^2 - 2x + 1 \\
x^2 + x - 6 &= 0 \\
(x+3)(x-2) &= 0
\end{aligned}$$

so $x = -3$ or $x = 2$. However, $x = 2$ results in a negative argument in the second logarithm of the original equation, so the lone solution is $x = -3$.

(c) We have

$$\begin{aligned}
\log_2(x) - \log_2(2x-5) &= 3 - \log_2(x+3) \\
\log_2\left(\frac{x}{2x-5}\right) + \log_2(x+3) &= 3 \\
\log_2\left(\frac{x(x+3)}{2x-5}\right) &= 3 \\
\frac{x(x+3)}{2x-5} &= 2^3 \\
\frac{x(x+3)}{2x-5} &= 8 \\
x(x+3) &= 8(2x-5) \\
x^2+3x &= 16x-40 \\
x^2-13x+40 &= 0 \\
(x-5)(x-8) &= 0
\end{aligned}$$

so $x = 5$ or $x = 8$. Both of these solutions are valid upon checking.

6. (a) We have

$$\begin{aligned}
6^{x-1} &= 3 \\
\ln\left(6^{x-1}\right) &= \ln(3) \\
(x-1)\ln(6) &= \ln(3) \\
x-1 &= \frac{\ln(3)}{\ln(6)} \\
x &= 1 + \frac{\ln(3)}{\ln(6)}.
\end{aligned}$$

(b) We have

$$\begin{aligned}
5^{2x+1} &= 3 \cdot 4^{-x} \\
\ln\left(5^{2x+1}\right) &= \ln\left(3 \cdot 4^{-x}\right) \\
\ln\left(5^{2x+1}\right) &= \ln(3) + \ln\left(4^{-x}\right) \\
(2x+1)\ln(5) &= \ln(3) - x\ln(4) \\
2x\ln(5) + x\ln(4) &= \ln(3) - \ln(5) \\
x[2\ln(5) + \ln(4)] &= \ln(3) - \ln(5) \\
x &= \frac{\ln(3)-\ln(5)}{2\ln(5)+\ln(4)} = \frac{\ln(3)-\ln(5)}{\ln(100)}.
\end{aligned}$$

(c) If we replace $e^x$ with, say, $t$ then this shows that the given equation is comparable to the quadratic equation

$$t^2 = 3t + 4.$$

So let's try solving it in exactly the same way:

$$\begin{aligned} e^{2x} &= 3e^x + 4 \\ (e^x)^2 &= 3e^x + 4 \\ (e^x)^2 - 3e^x - 4 &= 0 \\ (e^x - 4)(e^x + 1) &= 0 \end{aligned}$$

and therefore $e^x = 4$ or $e^x = -1$. If $e^x = 4$ then $x = \ln(4)$. It is not possible that $e^x = -1$, since the range of any exponential function is the interval $(0, \infty)$. Hence the only solution is $x = \ln(4)$.