MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SOLUTIONS

1. We have

$$\begin{aligned}(f \circ g)(x) = f(g(x)) &= f(x^2 + 2x + 1)\\ &= 3(x^2 + 2x + 1) - 5\\ &= 3x^2 + 6x + 3 - 5\\ &= 3x^2 + 6x - 2\end{aligned}$$

and

$$\begin{aligned}(g \circ f)(x) = g(f(x)) &= g(3x - 5)\\ &= (3x - 5)^2 + 2(3x - 5) + 1\\ &= 9x^2 - 30x + 25 + 6x - 10 + 1\\ &= 9x^2 - 24x + 16.\end{aligned}$$

2. We compute

$$\begin{aligned}(f \circ g)(x) = f(g(x)) &= f\left(\sqrt{2x-1}\right)\\ &= 5\left(\sqrt{2x-1}\right)^2 + 3\\ &= 5(2x - 1) + 3\\ &= 10x - 2.\end{aligned}$$

This function is always defined, but $g(x)$ is only defined if $2x - 1 \geq 0$, so $x \geq \frac{1}{2}$. Therefore

$$D_{f \circ g} = \left[\frac{1}{2}, \infty\right).$$

3. (a) Here, the polynomial is "inside" the square root, so we can set

$$f(x) = \sqrt{x} \quad \text{and} \quad g(x) = 3x^3 - 5.$$

(b) In this case, the linear function is inside the power of three in the denominator, so one solution is

$$f(x) = \frac{4}{x^3} \quad \text{and} \quad g(x) = 1 - 5x.$$

4. (a) We set

$$\begin{aligned} x &= \frac{1}{3}y + 2 \\ \frac{1}{3}y &= x - 2 \\ y &= 3x - 6. \end{aligned}$$

Note that $D_f = R_f = \mathbb{R}$, so $D_{f^{-1}} = R_{f^{-1}} = \mathbb{R}$. Hence

$$f^{-1}(x) = 3x - 6.$$

(b) We have

$$\begin{aligned} x &= \sqrt{y} - 3 \\ \sqrt{y} &= x + 3 \\ y &= (x+3)^2 \\ &= x^2 + 6x + 9. \end{aligned}$$

Also, $D_{f^{-1}} = R_f = [-3, \infty)$ while $R_{f^{-1}} = D_f = [0, \infty)$. Hence

$$f^{-1}(x) = x^2 + 6x + 9, x \geq -3.$$

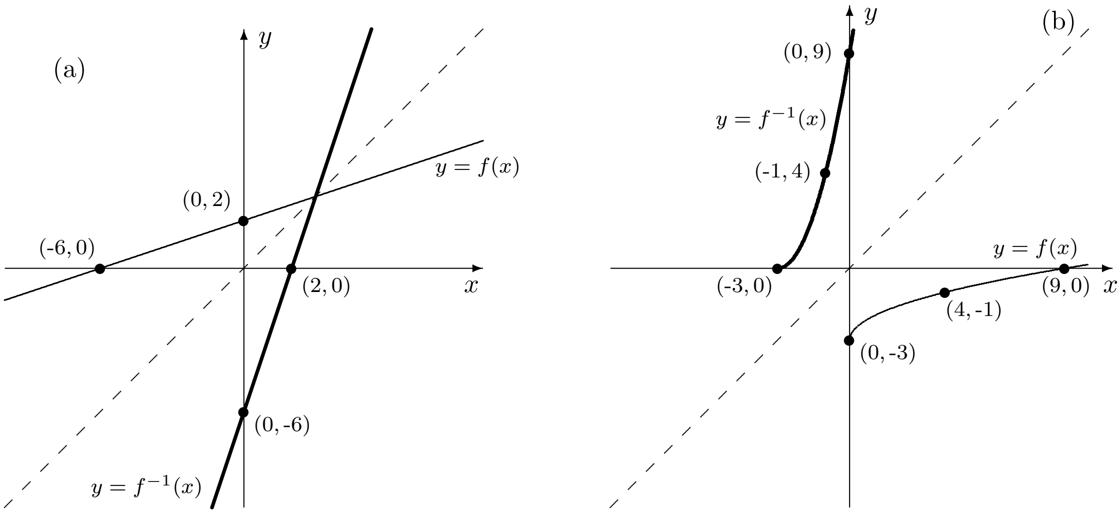
(c) Finally,

$$\begin{aligned} x &= 2\sqrt{y+1} \\ \sqrt{y+1} &= \frac{1}{2}x \\ y + 1 &= \frac{1}{4}x^2 \\ y &= \frac{1}{4}x^2 - 1. \end{aligned}$$

In this case, $D_{f^{-1}} = R_f = [0, \infty)$ while $R_{f^{-1}} = D_f = [-1, \infty)$. So now

$$f^{-1}(x) = \frac{1}{4}x^2 - 1, x \geq 0.$$

5. (a) To graph $f(x)$, we first recognise that this is a line, so we just need the x- and y-intercepts. Setting $\frac{1}{3}x + 2 = 0$, we see that $x = -6$ so $(-6, 0)$ is the x-intercept. Also, if $x = 0$ then $y = \frac{1}{3} \cdot 0 + 2 = 2$ so $(0, 2)$ is the y-intercept. But the reflection through $y = x$ of a line is still a line, and we can obtain two points on this line just by interchanging the coordinates of the intercepts of $f(x)$. In other words, $(0, -6)$ and $(2, 0)$ must be points on the graph of $f^{-1}(x)$, and so we can sketch this graph as well.



(b) Since $f(x)$ is a square root function, we can immediately note that its vertex is $(0, -3)$, which is also its y-intercept. To check for an x-intercept, we set $\sqrt{x} - 3 = 0$ which implies that $x = 9$, so $(9, 0)$ is the x-intercept. We need one more point on the graph, such as $(4, -1)$. The inverse function will therefore have a vertex at $(-3, 0)$ and the points $(-1, 4)$ and $(0, 9)$ will lie on its graph.

6. (a) Observe that if

$$\begin{aligned} f(a) &= f(b) \\ a^2 + 4 &= b^2 + 4 \\ a^2 &= b^2 \\ a &= \pm b. \end{aligned}$$

Hence we can pick any pair of arithmetic inverses to show that $f(x)$ is not one-to-one. For instance, $f(1) = f(-1) = 5$ while $f(4) = f(-4) = 20$.

(b) We have

$$\begin{aligned} x &= y^2 + 4 \\ y^2 &= x - 4 \\ y &= \pm\sqrt{x - 4}. \end{aligned}$$

But $R_{f^{-1}} = D_f = [0, \infty)$ so y here must be positive. Thus we can choose the positive square root only. Finally, $D_{f^{-1}} = R_f = [4, \infty)$ so

$$f^{-1}(x) = \sqrt{x - 4}, x \geq 4.$$

(c) We have

$$\begin{aligned} x &= y^2 + 4 \\ y^2 &= x - 4 \\ y &= \pm\sqrt{x - 4}. \end{aligned}$$

But this time, $R_{f^{-1}} = D_f = (-\infty, 0]$ so y here must be negative. Thus we can choose the negative square root only. Since $D_{f^{-1}} = R_f = [4, \infty)$, we conclude that

$$f^{-1}(x) = -\sqrt{x-4}, x \geq 4.$$