MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 3 | **Math 1090** | FALL 2009

SOLUTIONS

[3] 1. (a) We can write this function as

$$y = \sqrt{-(x-5)},$$

and see that its graph undergoes both a horizontal reflection and a horizontal translation 5 units to the right.

[3] (b) The graph of this function undergoes a vertical reflection, a vertical translation 2 units upwards, and a vertical skewing (specifically, a vertical stretching).

[3] (c) $x = -3$

[3] (d) $160 \cdot \frac{\pi}{180} = \frac{8\pi}{9}$

[3] (e) We know that sine is negative in the third and fourth quadrants. Tangent, and therefore cotangent, is negative in the second and fourth quadrants. Thus θ must lie in the fourth quadrant.

[3] 2. (a) $\theta = 30^\circ = \frac{\pi}{6}$

[3] (b) We can obtain $\tan(\theta) = \sqrt{3}$ by having $\sin(\theta) = \frac{\sqrt{3}}{2}$ and $\cos(\theta) = \frac{1}{2}$, because

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}.$$

We obtain these trig values for $\theta = 60^\circ = \frac{\pi}{3}$.

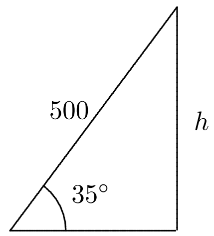
[3] (c) Let θ_r be the reference angle of θ. Then we know that $\cos(\theta_r) = \frac{\sqrt{3}}{2}$ as well, and therefore $\theta_r = 30^\circ$. The angle in the fourth quadrant whose reference angle is 30° is $\theta = 330^\circ = \frac{11\pi}{6}$.

(d) *(This question was omitted from the marking scheme due to a typographical error. The test paper originally identified θ as being in the third quadrant, not the second.)*

Let θ_r be the reference angle of θ. Then we know that $\tan(\theta_r) = 1$. This happens when $\sin(\theta_r) = \cos(\theta_r)$, which suggests that $\theta_r = 45^\circ$. The angle in the second quadrant whose reference angle is 45° is $\theta = 135^\circ = \frac{3\pi}{4}$.

[3] (e) $\theta = 90^\circ = \frac{\pi}{2}$

[4] 3. We draw a diagram:

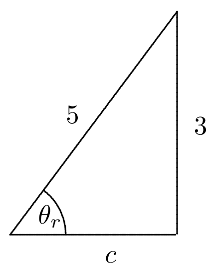


We want to solve for the height h of the hill. Observe that

$$\begin{aligned} \sin(35^\circ) &= \frac{h}{500} \\ h &= 500\sin(35^\circ) \\ &\approx 500(0.574) \\ &\approx 287. \end{aligned}$$

We conclude that the hill is about 287 metres high.

[6] 4. Let θ_r be the reference angle for θ, so $\sin(\theta_r) = \dfrac{3}{5}$ as well. We draw a diagram:



By the Pythagorean theorem,

$$\begin{aligned} c^2 &= 5^2 - 3^2 = 16 \\ c &= \sqrt{16} = 4. \end{aligned}$$

Thus $\cos(\theta_r) = \dfrac{4}{5}$ and $\tan(\theta_r) = \frac{3}{4}$.

However, θ is in the second quadrant, where cosine and tangent are both negative. This means that

$$\cos(\theta) = -\frac{4}{5} \quad \text{and} \quad \tan(\theta) = -\frac{3}{4}.$$

Finally, taking reciprocals, we have

$$\sec(\theta) = -\frac{5}{4}, \quad \csc(\theta) = \frac{5}{3}, \quad \cot(\theta) = -\frac{4}{3}.$$

[5] 5. We begin from the left, where we can write

$$\frac{1}{\cos(x)\csc^2(x)} = \frac{\sin^2(x)}{\cos(x)}.$$

Recall that $\sin^2(x) + \cos^2(x) = 1$ so $\sin^2(x) = 1 - \cos^2(x)$. Thus

$$\begin{aligned}\frac{1}{\cos(x)\csc^2(x)} &= \frac{1-\cos^2(x)}{\cos(x)} \\ &= \frac{1}{\cos(x)} - \cos(x) \\ &= \sec(x) - \cos(x),\end{aligned}$$

as desired.