MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 3.5	**Math 1090**	Fall 2009

SOLUTIONS

1. (a) We have

$$\begin{aligned} \frac{\cot^2(x)}{3} &= 1 \\ \cot^2(x) &= 3 \\ \cot(x) &= \pm\sqrt{3} \\ \tan(x) &= \pm\frac{\sqrt{3}}{3}. \end{aligned}$$

(It's not strictly necessary to turn the cotangent into a tangent function, but people typically become more familiar with the values of tangent than with cotangent.) If $\tan(x) = \frac{\sqrt{3}}{3}$ then the solutions are $x = \frac{\pi}{6}$ and $x = \frac{7\pi}{6}$. If $\tan(x) = -\frac{\sqrt{3}}{3}$ then the solutions are $x = \frac{5\pi}{6}$ and $x = \frac{11\pi}{6}$. Hence this equation has four solutions:

$$x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}, x = \frac{7\pi}{6}, \quad x = \frac{11\pi}{6}.$$

(b) We write

$$\begin{aligned} \sin^2(x) &= \frac{1}{2}\sin(x) \\ \sin^2(x) - \frac{1}{2}\sin(x) &= 0 \\ \sin(x)\left[\sin(x) - \frac{1}{2}\right] &= 0 \end{aligned}$$

so either $\sin(x) = 0$ (and thus $x = 0$ or $x = \pi$) or $\sin(x) = \frac{1}{2}$ (which means $x = \frac{\pi}{6}$ or $x = \frac{5\pi}{6}$). Thus the four solutions are

$$x = 0, \quad x = \frac{\pi}{6}, \quad x = \frac{5\pi}{6}, \quad x = \pi.$$

(c) We have

$$\begin{aligned} \sin^2(x) &= \cos(x)\sin(x) \\ \sin^2(x) - \cos(x)\sin(x) &= 0 \\ \sin(x)[\sin(x) - \cos(x)] &= 0. \end{aligned}$$

Clearly, we can have $\sin(x) = 0$ so $x = 0$ or $x = \pi$. Otherwise, this results in $\sin(x) = \cos(x)$ which is not a particularly helpful equation: we want to get a trig function equal to some familiar number. However, if we divide both sides of this by $\cos(x)$, we arrive at $\tan(x) = 1$, so now we see that $x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$. Again, then, we have four solutions:

$$x = 0, \quad x = \frac{\pi}{4}, \quad x = \pi, \quad x = \frac{5\pi}{4}.$$

(d) We factor as we would for a quadratic equation:

$$\begin{aligned} 2\sin^2(x) - \sin(x) - 1 &= 0 \\ [2\sin(x) + 1][\sin(x) - 1] &= 0 \end{aligned}$$

so either $\sin(x) = -\frac{1}{2}$ (and therefore $x = \frac{7\pi}{6}$ or $x = \frac{11\pi}{6}$), or $\sin(x) = 1$ (so $x = \frac{\pi}{2}$). Hence this equation has three solutions:

$$x = \frac{\pi}{2}, \quad x = \frac{7\pi}{6}, \quad \frac{11\pi}{6}.$$

[5] (e) Since $\sec^2(x) = \tan^2(x) + 1$, we have

$$\begin{aligned} \sec^2(x) + \tan^2(x) &= 7 \\ [\tan^2(x) + 1] + \tan^2(x) &= 7 \\ 2\tan^2(x) &= 6 \\ \tan^2(x) &= 3 \\ \tan(x) &= \pm\sqrt{3}. \end{aligned}$$

If $\tan(x) = \sqrt{3}$ then $x = \frac{\pi}{3}$ or $x = \frac{4\pi}{3}$. If $\tan(x) = -\sqrt{3}$ then $x = \frac{2\pi}{3}$ or $x = \frac{5\pi}{3}$. Thus the four solutions are

$$x = \frac{\pi}{3}, \quad x = \frac{2\pi}{3}, \quad x = \frac{4\pi}{3}, \quad x = \frac{5\pi}{3}.$$