MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.4 **Math 1090** FALL 2009

SOLUTIONS

1. (a) We begin with the lefthand side:

$$\begin{aligned}\sin^2(t)[1+\cot^2(t)] &= \sin^2(t)\left[1+\frac{\cos^2(t)}{\sin^2(t)}\right]\\ &= \sin^2(t)+\cos^2(t)\\ &= 1.\end{aligned}$$

(b) We start on the left:

$$\frac{\tan(\theta)\cot(\theta)}{\csc(\theta)} = \frac{1}{\csc(\theta)} = \sin(\theta).$$

(c) We begin on the righthand side this time:

$$\begin{aligned}\sec^2(x)+\csc^2(x) &= \frac{1}{\cos^2(x)}+\frac{1}{\sin^2(x)}\\ &= \frac{\sin^2(x)+\cos^2(x)}{\cos^2(x)\sin^2(x)}\\ &= \frac{1}{\cos^2(x)\sin^2(x)}\\ &= \frac{1}{\cos^2(x)}\cdot\frac{1}{\sin^2(x)}\\ &= \sec^2(x)\csc^2(x).\end{aligned}$$

(d) We start on the left:

$$\begin{aligned}\frac{\sin(x)}{1+\cos(x)}-\frac{1-\cos(x)}{\sin(x)} &= \frac{\sin^2(x)-[1-\cos(x)][1+\cos(x)]}{\sin(x)[1+\cos(x)]}\\ &= \frac{\sin^2(x)+\cos^2(x)-1}{\sin(x)[1+\cos(x)]}\\ &= \frac{1-1}{\sin(x)[1+\cos(x)]}\\ &= 0.\end{aligned}$$

(e) We start on the right:

$$\begin{aligned}
[\sec(\theta) - \tan(\theta)]^2 &= \left[\frac{1}{\cos(\theta)} - \frac{\sin(\theta)}{\cos(\theta)}\right]^2 \\
&= \left[\frac{1 - \sin(\theta)}{\cos(\theta)}\right]^2 \\
&= \frac{[1 - \sin(\theta)]^2}{\cos^2(\theta)} \\
&= \frac{[1 - \sin(\theta)]^2}{1 - \sin^2(\theta)} \\
&= \frac{[1 - \sin(\theta)]^2}{[1 - \sin(\theta)][1 + \sin(\theta)]} \\
&= \frac{1 - \sin(\theta)}{1 + \sin(\theta)}.
\end{aligned}$$