MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTIONS 3.2 & 3.3 **Math 1090** FALL 2009

SOLUTIONS

1. (a) $\dfrac{7\pi}{12} \cdot \dfrac{180^\circ}{\pi} = 105^\circ$

   (b) $225 \cdot \dfrac{\pi}{180} = \dfrac{5\pi}{4}$

   (c) $-40 \cdot \dfrac{\pi}{180} = -\dfrac{2\pi}{9}$

2. (a) Observe that

$$\frac{17\pi}{6} = \frac{5\pi}{6} + 2\pi$$

so $\frac{17\pi}{6}$ behaves exactly like $\frac{5\pi}{6}$. It lies in the second quadrant, where only sine (and therefore cosecant) are positive. Its reference angle is $\frac{\pi}{6}$, so the trig values of these two angles will be the same, except possibly for their signs. Thus

$$\sin\left(\frac{17\pi}{6}\right) = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$$

$$\cos\left(\frac{17\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2}$$

$$\tan\left(\frac{17\pi}{6}\right) = -\tan\left(\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{3}$$

$$\cot\left(\frac{17\pi}{6}\right) = -\cot\left(\frac{\pi}{6}\right) = -\sqrt{3}$$

$$\sec\left(\frac{17\pi}{6}\right) = -\sec\left(\frac{\pi}{6}\right) = -\frac{2\sqrt{3}}{3}$$

$$\csc\left(\frac{17\pi}{6}\right) = \csc\left(\frac{\pi}{6}\right) = 2.$$

   (b) The reference angle for $-\frac{3\pi}{4}$ is $\frac{\pi}{4}$, and $-\frac{3\pi}{4}$ is in the third quadrant, where only tangent (and therefore cotangent) are positive. Thus

$$\begin{aligned}
\sin\left(-\frac{3\pi}{4}\right) &= -\sin\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \\
\cos\left(-\frac{3\pi}{4}\right) &= -\cos\left(\frac{\pi}{4}\right) = -\frac{\sqrt{2}}{2} \\
\tan\left(-\frac{3\pi}{4}\right) &= \tan\left(\frac{\pi}{4}\right) = 1 \\
\cot\left(-\frac{3\pi}{4}\right) &= \cot\left(\frac{\pi}{4}\right) = 1 \\
\sec\left(-\frac{3\pi}{4}\right) &= -\sec\left(\frac{\pi}{4}\right) = -\sqrt{2} \\
\csc\left(-\frac{3\pi}{4}\right) &= -\csc\left(\frac{\pi}{4}\right) = -\sqrt{2}.
\end{aligned}$$

(c) Since

$$\frac{5\pi}{2} = \frac{\pi}{2} + 2\pi,$$

it will have the same trig values as $\frac{\pi}{2}$:

$$\begin{aligned}
\sin\left(\frac{5\pi}{2}\right) &= \sin\left(\frac{\pi}{2}\right) = 1 \\
\cos\left(\frac{5\pi}{2}\right) &= \cos\left(\frac{\pi}{2}\right) = 0 \\
\cot\left(\frac{5\pi}{2}\right) &= \cot\left(\frac{\pi}{2}\right) = 0 \\
\csc\left(\frac{5\pi}{2}\right) &= \csc\left(\frac{\pi}{2}\right) = 1.
\end{aligned}$$

Note that both $\tan\left(\frac{5\pi}{2}\right)$ and $\sec\left(\frac{5\pi}{2}\right)$ are undefined.

3. Cosine is positive in the first and fourth quadrants, while cosecant is negative in the third and fourth quadrants. Thus θ must lie in the fourth quadrant.

4. (a) The angle $\theta + \pi$ is exactly half a revolution around the unit circle from the angle θ. This means that θ and $\theta + \pi$ will have the same reference angle. (If this isn't obvious to you, draw some representative angles θ and $\theta + \pi$.) Furthermore, we know that θ must be in either the first or the fourth quadrant (since its cosine is positive), which means that $\theta + \pi$ will be in either the second or the third quadrant, where cosine is negative. Thus

$$\cos(\theta + \pi) = -\frac{2}{5}.$$

(b) The angle $\theta + 2\pi$ is a full revolution around the unit circle from the angle θ, and so they have the same trig values. Consequently,

$$\cos(\theta + 2\pi) = \frac{2}{5}.$$

5. (a) We construct a right triangle with opposite sidelength 8 and hypotenuse 17; by the Pythagorean theorem, the adjacent sidelength is

$$\sqrt{17^2 - 8^2} = \sqrt{289 - 64} = \sqrt{225} = 15.$$

Since θ is in the second quadrant, only sine and cosecant are positive. Thus

$$\begin{aligned}
\cos(\theta) &= -\frac{15}{17} \\
\tan(\theta) &= -\frac{8}{15} \\
\cot(\theta) &= -\frac{15}{8} \\
\sec(\theta) &= -\frac{17}{15} \\
\csc(\theta) &= \frac{17}{8}.
\end{aligned}$$

(b) We construct a right triangle with adjacent sidelength 1 and hypotenuse of length 3; by the Pythagorean theorem, the opposite sidelength is

$$\sqrt{3^2 - 1^2} = \sqrt{9 - 1} = \sqrt{8} = 2\sqrt{2}.$$

Since θ is in the third quadrant, only tangent and cotangent are positive. Thus

$$\begin{aligned}
\sin(\theta) &= -\frac{2\sqrt{2}}{3} \\
\tan(\theta) &= \frac{2\sqrt{2}}{1} = 2\sqrt{2} \\
\cot(\theta) &= \frac{1}{2\sqrt{2}} = \frac{\sqrt{2}}{4} \\
\sec(\theta) &= -3 \\
\csc(\theta) &= -\frac{3}{2\sqrt{2}} = -\frac{3\sqrt{2}}{4}.
\end{aligned}$$

(c) We construct a right triangle with hypotenuse of length 5 and adjacent sidelength 3; by the Pythagorean theorem, the opposite sidelength is

$$\sqrt{5^2 - 3^2} = \sqrt{25 - 9} = \sqrt{16} = 4.$$

Since θ is in the fourth quadrant, only cosine and secant are positive, so we see that

$$\begin{aligned}
\sin(\theta) &= -\frac{4}{5} \\
\cos(\theta) &= \frac{3}{5} \\
\tan(\theta) &= -\frac{4}{3} \\
\cot(\theta) &= -\frac{3}{4} \\
\csc(\theta) &= -\frac{5}{4}.
\end{aligned}$$