MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 3.1 **Math 1090** FALL 2009

SOLUTIONS

1. (a) We already know that

$$\sin(0^\circ) = 0 \quad \text{and} \quad \cos(0^\circ) = 1.$$

This means that

$$\tan(0^\circ) = \frac{\sin(0^\circ)}{\cos(0^\circ)} = \frac{0}{1} = 0$$
$$\sec(0^\circ) = \frac{1}{\cos(0^\circ)} = \frac{1}{1} = 1$$
$$\csc(0^\circ) = \frac{1}{\sin(0^\circ)} = \frac{1}{0} \quad \text{(undefined)}$$
$$\cot(0^\circ) = \frac{1}{\tan(0^\circ)} = \frac{1}{0} \quad \text{(undefined)}.$$

(b) We already know that

$$\sin(30^\circ) = \frac{1}{2} \quad \text{and} \quad \cos(30^\circ) = \frac{\sqrt{3}}{2}.$$

This means that

$$\tan(30^\circ) = \frac{\sin(30^\circ)}{\cos(30^\circ)} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{2} \cdot \frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$
$$\sec(30^\circ) = \frac{1}{\cos(30^\circ)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$
$$\csc(30^\circ) = \frac{1}{\sin(30^\circ)} = \frac{1}{\frac{1}{2}} = 2$$
$$\cot(30^\circ) = \frac{1}{\tan(30^\circ)} = \frac{1}{\frac{\sqrt{3}}{3}} = \frac{3}{\sqrt{3}} = \sqrt{3}.$$

(c) We already know that

$$\sin(45^\circ) = \cos(45^\circ) = \frac{\sqrt{2}}{2}.$$

This means that

$$\tan(45°) = \frac{\sin(45°)}{\cos(45°)} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{2}}{2}} = 1$$
$$\sec(45°) = \frac{1}{\cos(45°)} = \frac{1}{\frac{\sqrt{2}}{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$$
$$\csc(45°) = \frac{1}{\sin(45°)} = \frac{1}{\frac{\sqrt{2}}{2}} = \sqrt{2}$$
$$\cot(45°) = \frac{1}{\tan(45°)} = \frac{1}{1} = 1.$$

(d) We already know that

$$\sin(60°) = \frac{\sqrt{3}}{2} \quad \text{and} \quad \cos(60°) = \frac{1}{2}.$$

This means that

$$\tan(60°) = \frac{\sin(60°)}{\cos(60°)} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{\sqrt{3}}{2} \cdot 2 = \sqrt{3}$$
$$\sec(60°) = \frac{1}{\cos(60°)} = \frac{1}{\frac{1}{2}} = 2$$
$$\csc(60°) = \frac{1}{\sin(60°)} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$
$$\cot(60°) = \frac{1}{\tan(60°)} = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

(e) We already know that

$$\sin(90°) = 1 \quad \text{and} \quad \cos(90°) = 0.$$

This means that

$$\tan(90°) = \frac{\sin(90°)}{\cos(90°)} = \frac{1}{0} \quad \text{(undefined)}$$
$$\sec(90°) = \frac{1}{\cos(90°)} = \frac{1}{0} \quad \text{(undefined)}$$
$$\csc(90°) = \frac{1}{\sin(90°)} = \frac{1}{1} = 1$$
$$\cot(90°) = \frac{1}{\tan(90°)} = \frac{1}{\frac{1}{0}} = \frac{0}{1} = 0.$$

2. There are several approaches we could take. First, we could simply observe that
$$\tan(\theta) = \frac{4\sqrt{3}}{4} = \sqrt{3},$$
and see from #1(d), above, that $\theta = 60°$.

   On the other hand, if you're more comfortable working with values of sine and cosine, we can use the Pythagorean theorem to find the length h of the hypotenuse:
$$h^2 = \left(4\sqrt{3}\right)^2 + 4^2 = 48 + 16 = 64$$
$$h = \sqrt{64} = 8.$$
   So then, for instance,
$$\cos(\theta) = \frac{4}{8} = \frac{1}{2},$$
   which you should immediately recognise as the cosine of 60°.

3. We construct a right triangle with interior angle θ, opposite sidelength 12 and adjacent sidelength 5; by the Pythagorean theorem, the length h of the hypotenuse is
$$h^2 = 12^2 + 5^2 = 144 + 25 = 169$$
$$h = \sqrt{169} = 13.$$
   Now we can see that
$$\sin(\theta) = \frac{12}{13}$$
$$\cos(\theta) = \frac{5}{13}$$
$$\cot(\theta) = \frac{5}{12}$$
$$\sec(\theta) = \frac{13}{5}$$
$$\csc(\theta) = \frac{13}{12}.$$

4. If we envision this as a right triangle, then the length of the side opposite the 35° angle is 4, and we're trying to find the length of the hypotenuse, h. An appropriate trig ratio to use, then, is the sine function:
$$\sin(35°) = \frac{4}{h}$$
$$h = \frac{4}{\sin(35°)}$$
$$\approx \frac{4}{0.57358}$$
$$\approx 6.97379.$$
   To two decimal places, therefore, the ladder is approximately 6.97 metres long.