MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 2 **Math 1090** FALL 2009

SOLUTIONS

[3] 1. (a) Given a linear, absolute value, or quadratic function $f(x)$, we can let x be any real number and $f(x)$ will be defined, so their domain $D = \mathbb{R}$. For square root functions, this is not true: any value of x which makes the expression under the square root negative will cause $f(x)$ to be undefined. Thus square root functions are the only function on this list whose domain is not $\mathbb{R}$.

[3] (b) The graphs of absolute value functions, quadratic functions and square root functions can all lie entirely above (or below) the x-axis, meaning that they would have no x-intercepts. Another way to think about this is to observe that if $f(x)$ is an absolute value, quadratic or square root function then it is possible for the equation $f(x) = 0$ to have no solutions. This is not true if $f(x)$ is a linear function, because the linear equation $f(x) = 0$ always possesses exactly one solution.

[3] (c) If $f(x)$ is a linear, absolute value or quadratic function, then $f(x)$ is defined for all x, and certainly for $x = 0$. This means that all these functions have a y-intercept. The only type of function on this list for which $f(0)$ may not be defined — and which therefore may not have a y-intercept — is the square root function.

[3] (d) As suggested by the discussion for part (b), above, we also know that if $f(x)$ is an absolute value, quadratic or square root function then the equation $f(x) = 0$ might possesses two solutions. This means that these types of functions can have two x-intercepts. Again, if $f(x)$ is a linear function then $f(x) = 0$ has exactly one solution, and therefore always has one, and only one, x-intercept.

[3] (e) If a function possesses two y-intercepts, there would be two points $(0, p)$ and $(0, q)$ on the graph. But then the graph would fail the Vertical Line Test, meaning that it doesn't represent a function at all. Thus no function can have two y-intercepts.

[4] 2. (a) Observe that

$$9x^2 - 12x - 5 = (3x - 5)(3x + 1).$$

To find the x-intercepts, we set

$$(3x - 5)(3x + 1) = 0,$$

which means that either $3x - 5 = 0$, so $x = \frac{5}{3}$, or $3x + 1 = 0$, so $x = -\frac{1}{3}$. Thus the x-intercepts are the points $\left(\frac{5}{3}, 0\right)$ and $\left(-\frac{1}{3}, 0\right)$.

[4] (b) We factor out the 9 to obtain

$$f(x) = 9\left[x^2 - \frac{4}{3}x - \frac{5}{9}\right] = 9\left[\left(x^2 - \frac{4}{3}x\right) - \frac{5}{9}\right].$$

The coefficient of x is $-\frac{4}{3}$, so we set $p = \frac{1}{2}\left(-\frac{4}{3}\right) = -\frac{2}{3}$. Thus $q = p^2 = \left(-\frac{2}{3}\right)^2 = \frac{4}{9}$. Adding p inside the round brackets and subtracting it outside, we have

$$\begin{aligned} f(x) &= 9\left[\left(x^2 - \frac{4}{3}x + \frac{4}{9}\right) - \frac{5}{9} - \frac{4}{9}\right] \\ &= 9\left[\left(x - \frac{2}{3}\right)^2 - 1\right] \\ &= 9\left(x - \frac{2}{3}\right)^2 - 9. \end{aligned}$$

Now we can see that the vertex is the point $\left(\frac{2}{3}, -9\right)$.

[5] 3. We need to consider two possibilities. First it may be that $|2x+5| = 2x+5$. Then the equation becomes

$$\begin{aligned} 3x + (2x+5) &= 0 \\ 5x &= -5 \\ x &= -1. \end{aligned}$$

On the other hand, it may be that $|2x+5| = -(2x+5)$. Now we have

$$\begin{aligned} 3x - (2x+5) &= 0 \\ x - 5 &= 0 \\ x &= 5. \end{aligned}$$

We need to check these solutions to see if they're valid. For $x = -1$, we substitute back into the original equation to obtain

$$3(-1) + |2(-1)+5| = -3 + |-2+5| = -3 + |3| = -3 + 3 = 0,$$

as required. For $x = 5$, we get

$$3 \cdot 5 + |2 \cdot 5 + 5| = 15 + |15| = 15 + 15 = 30 \neq 0.$$

This means that $x = 5$ is not a solution; only $x = -1$ is a solution of the equation.

[4] 4. We multiply by the conjugate of the denominator, obtaining

$$\begin{aligned}
\frac{8}{4-5\sqrt{2}} \cdot \frac{4+5\sqrt{2}}{4+5\sqrt{2}} &= \frac{8\left(4+5\sqrt{2}\right)}{4^2-\left(5\sqrt{2}\right)^2} \\
&= \frac{8\left(4+5\sqrt{2}\right)}{16-25\cdot 2} \\
&= \frac{8\left(4+5\sqrt{2}\right)}{16-50} \\
&= \frac{8\left(4+5\sqrt{2}\right)}{-34} \\
&= -\frac{4\left(4+5\sqrt{2}\right)}{17}.
\end{aligned}$$

[8] 5. This is a square root function, so we know its graph is a semi-parabola. Let's first write it in standard form, as

$$f(x) = -\sqrt{x+1} + 2.$$

Now we can see that the vertex is the point $(-1, 2)$.

To find the x-intercept, we set

$$\begin{aligned}
2 - \sqrt{x+1} &= 0 \\
2 &= \sqrt{x+1} \\
2^2 &= \left(\sqrt{x+1}\right)^2 \\
4 &= x+1 \\
3x
\end{aligned}$$

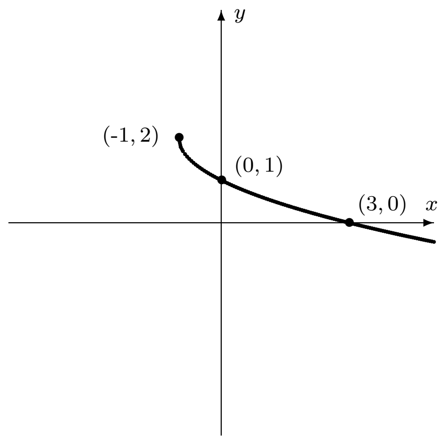
so there is one x-intercept, namely the point $(3, 0)$.

Next, observe that

$$f(0) = 2 - \sqrt{0+1} = 2 - \sqrt{1} = 2 - 1 = 1,$$

so there is a y-intercept at $(0, 1)$.

Since we now have the vertex and two other points on the curve, we can immediately sketch the graph.



From this, we can see that the domain of the function is $D = [-1, \infty)$ and its range is $R = (-\infty, 2]$.