MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.4 **Math 1090** FALL 2009

SOLUTIONS

1. (a) Squaring both sides, we obtain

$$\begin{aligned} 2-x &= (2-x)^2 \\ 2-x &= 4-4x+x^2 \\ x^2-3x+2 &= 0 \\ (x-2)(x-1) &= 0 \end{aligned}$$

so the two possible solutions are $x = 2$ and $x = 1$. We need to check these solutions. For $x = 2$, we see that

$$\sqrt{2-x} = \sqrt{2-2} = 0 \quad \text{and} \quad 2-x = 2-2 = 0,$$

so this is a solution. For $x = 1$, we have

$$\sqrt{2-1} = \sqrt{1} = 1 \quad \text{and} \quad 2-1 = 1$$

so this is also a solution. Hence $x = 2$ and $x = 1$ are both solutions of the equation.

(b) We isolate the square root to get

$$\begin{aligned} \sqrt{4x-1} &= 3-4x \\ 4x-1 &= (3-4x)^2 \\ 4x-1 &= 9-24x+16x^2 \\ 16x^2-28x+10 &= 0 \\ 8x^2-14x+5 &= 0 \\ (4x-5)(2x-1) &= 0 \end{aligned}$$

so the two possible solutions are $x = \frac{5}{4}$ and $x = \frac{1}{2}$. However, substitution of $x = \frac{5}{4}$ into the original equation gives

$$\sqrt{4x-1} + 4x = 7 \neq 3$$

so only $x = \frac{1}{2}$ is a solution.

(c) Rearranging the equation, we have

$$\begin{aligned} \sqrt{5x-4} &= 1-3x \\ 5x-4 &= (1-3x)^2 \\ 5x-4 &= 9x^2-6x+1 \\ 9x^2-11x+5 &= 0. \end{aligned}$$

This cannot be factored, so we use the quadratic formula:

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4 \cdot 9 \cdot 5}}{2 \cdot 9} = \frac{11 \pm \sqrt{121 - 180}}{18} = \frac{11 \pm \sqrt{-59}}{18}.$$

We don't need to go any further because we can see now that the discriminant (-59) is negative, which means that there are no real solutions x. Hence this equation has no solutions.

2. (a) We solve the inequality

$$\begin{aligned} 3 - 7x &\geq 0 \\ 3 &\geq 7x \\ \frac{3}{7} &\geq x. \end{aligned}$$

Hence the domain is $\left(-\infty, \frac{3}{7}\right]$.

(b) We require

$$\begin{aligned} 2x + 6 &\geq 0 \\ 2x &\geq -6 \\ x &\geq -3. \end{aligned}$$

Now we see that the domain is $[-3, \infty)$.

3. (a) Observe that

$$\sqrt{4x + 12} = \sqrt{4(x+3)} = \sqrt{4} \cdot \sqrt{x+3} = 2\sqrt{x+3}.$$

Thus

$$f(x) = \frac{1}{6} \cdot 2\sqrt{x+3} = \frac{1}{3}\sqrt{x+3}.$$

Comparing this to the standard form of a square root function, we see that $h = -3$ and $k = 0$, so the vertex is $(-3, 0)$.

(b) We can write

$$\sqrt{1 - 3x} = \sqrt{3\left(\frac{1}{3} - x\right)} = \sqrt{3} \cdot \sqrt{\frac{1}{3} - x}$$

so

$$g(x) = \sqrt{3} \cdot \sqrt{\frac{1}{3} - x} - 8.$$

(Remember that we cannot simplify $\sqrt{3}$ any further.) In the standard form of a square root function, $h = \frac{1}{3}$ and $k = -8$, so the vertex is the point $\left(\frac{1}{3}, -8\right)$.

4. (a) First note that the vertex is $(-4, 1)$. To find any x-intercepts, we set

$$\begin{aligned}\sqrt{x+4}+1 &= 0\\ \sqrt{x+4} &= -1,\end{aligned}$$

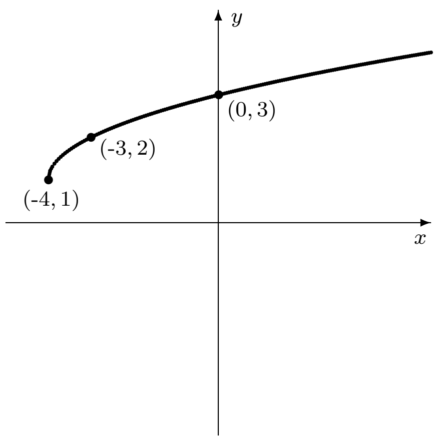
which is impossible because a square root can never be negative. Hence there is no x-intercept. To determine if there is a y-intercept, we set $x = 0$ and obtain

$$y = \sqrt{0+4}+1 = \sqrt{4}+1 = 3,$$

so $(0, 3)$ is the y-intercept. We need one more point to sketch the graph, such as $x = -3$, for which

$$y = \sqrt{-3+4}+1 = \sqrt{1}+1 = 2.$$

Hence $(-3, 2)$ is a point on the curve, and now we can sketch the graph.



We see that the domain of the function is $[-4, \infty)$ and the range is $[1, \infty)$.

(b) We rewrite the given equation as

$$y = -4\sqrt{x}+3$$

and see that the vertex is $(0, 3)$. This is also the y-intercept.
The x-intercept can be found by setting

$$\begin{aligned}-4\sqrt{x}+3 &= 0\\ 3 &= 4\sqrt{x}\\ \frac{3}{4} &= \sqrt{x}\\ \left(\frac{3}{4}\right)^2 &= x\\ \frac{9}{16} &= x.\end{aligned}$$

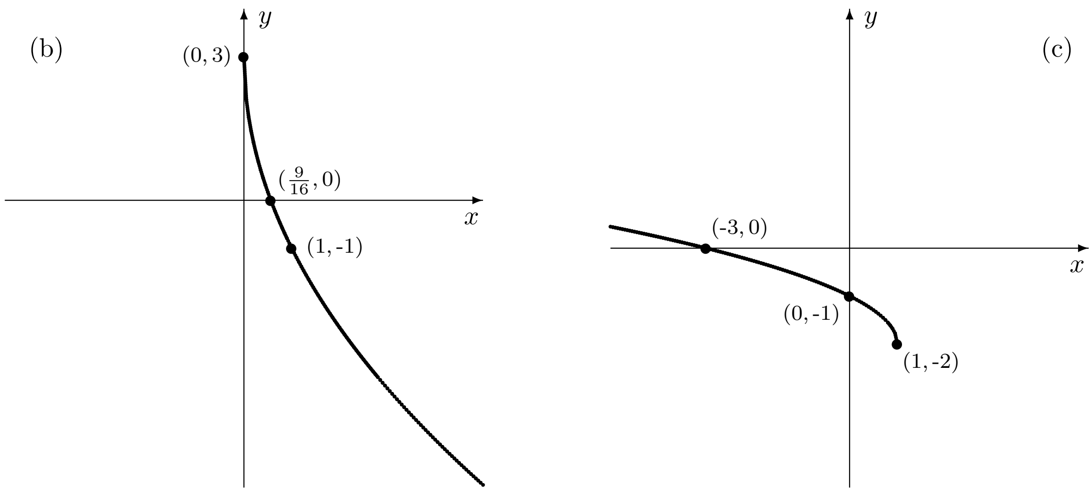
Observe that

$$-4\sqrt{\frac{9}{16}}+3=-4\cdot\frac{3}{4}+3=-3+3=0,$$

so this is a solution of the equation. Thus $\left(\frac{9}{16},0\right)$ is the x-intercept.
We need one more point on the graph, such as $x=1$, for which

$$y=-4\sqrt{1}+3=-4\cdot 1+3=-1,$$

so $(1,-1)$ is such a point. Now we can sketch the graph, below.
We see that the domain of the function is $[0,\infty)$ and the range is $(-\infty,3]$.



(c) We can immediately we see that the vertex is $(1,-2)$.
To find the x-intercept, we solve

$$\begin{aligned}\sqrt{1-x}-2&=0\\ \sqrt{1-x}&=2\\ 1-x&=2^2\\ 1-x&=4\\ -x&=3\\ x&=-3.\end{aligned}$$

Note that

$$\sqrt{1-(-3)}-2=\sqrt{4}-2=2-2=0,$$

so $(-3,0)$ is the x-intercept.
To find the y-intercept, we observe that

$$f(0)=\sqrt{1-0}-2=1-2=-1,$$

so $(0,-1)$ is the y-intercept. Now we can sketch the graph, above.
We see that the domain of the function is $(-\infty,1]$ and the range is $[-2,\infty)$.