MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Section 2.3 **Math 1090** Fall 2009

SOLUTIONS

1. $6\sqrt{20} - 5\sqrt{45} = 6\sqrt{4\cdot 5} - 5\sqrt{9\cdot 5} = 6(2)\sqrt{5} - 5(3)\sqrt{5} = 12\sqrt{5} - 15\sqrt{5} = -3\sqrt{5}$

2. (a) $\dfrac{15}{2\sqrt{5}} \cdot \dfrac{\sqrt{5}}{\sqrt{5}} = \dfrac{15\sqrt{5}}{2\cdot 5} = \dfrac{3\sqrt{5}}{2}$

   (b) $\dfrac{3}{6-4\sqrt{3}} \cdot \dfrac{6+4\sqrt{3}}{6+4\sqrt{3}} = \dfrac{3(6+4\sqrt{3})}{36-48} = \dfrac{3(6+4\sqrt{3})}{-12} = \dfrac{6+4\sqrt{3}}{-4} = -\dfrac{3+2\sqrt{3}}{2}$

   (c) $\dfrac{1}{\sqrt{7}+3\sqrt{2}} \cdot \dfrac{\sqrt{7}-3\sqrt{2}}{\sqrt{7}-3\sqrt{2}} = \dfrac{\sqrt{7}-3\sqrt{2}}{7-18} = -\dfrac{\sqrt{7}-3\sqrt{2}}{11}$

3. (a) $49x^2 - 16 = (7x)^2 - 4^2 = (7x-4)(7x+4)$

   (b) $x^2 - 7x + 6 = (x-6)(x-1)$

   (c) $x^2 - 5x + 6 = (x-3)(x-2)$

   (d) $x^2 + 5x - 24 = (x+8)(x-3)$

   (e) $4x^2 + 18x + 8 = (2x+8)(2x+1) = 2(x+4)(2x+1)$

   (f) $9x^2 - 30x + 25 = (3x-5)(3x-5) = (3x-5)^2$

   (g) $12x^2 - x - 6 = (4x-3)(3x+2)$

   (h) $4 + 19x - 5x^2 = -(5x^2 - 19x - 4) = -(5x+1)(x-4)$

4. (a) We have

$$\begin{aligned} 9x^2 - 16 &= 0 \\ 9x^2 &= 16 \\ x^2 &= \frac{16}{9} \\ x &= \pm\sqrt{\frac{16}{9}} \\ &= \pm\frac{4}{3}. \end{aligned}$$

   Hence the solutions are $x = \dfrac{4}{3}$ and $x = -\dfrac{4}{3}$.
   Alternatively, we could complete square immediately:

$$\begin{aligned} 9x^2 - 16 &= 0 \\ (3x-4)(3x+4) &= 0 \end{aligned}$$

   so either $3x - 4 = 0$, which means $x = \dfrac{4}{3}$, or $3x + 4 = 0$, which means $x = -\dfrac{4}{3}$.

(b) We have

$$\begin{aligned} 4x(7-x) &= 49 \\ 28x - 4x^2 &= 49 \\ 4x^2 - 28x + 49 &= 0 \\ (2x-7)^2 &= 0 \\ 2x - 7 &= 0 \end{aligned}$$

so $x = \frac{7}{2}$ is the only solution.

(c) Observe that

$$1 - 2x^2 - 4x = -2x^2 - 4x + 1.$$

Unfortunately, this cannot be factored, so we use the quadratic formula:

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(-2)(1)}}{2(-2)} = \frac{4 \pm \sqrt{16+8}}{-4} = \frac{4 \pm \sqrt{24}}{-4} = \frac{4 \pm 2\sqrt{6}}{-4}.$$

Thus the first solution is

$$x = \frac{4 + 2\sqrt{6}}{-4} = \frac{4}{-4} + \frac{2}{-4} = -1 - \frac{\sqrt{6}}{2}.$$

The second solution is

$$x = \frac{4 - 2\sqrt{6}}{-4} = \frac{4}{-4} - \frac{2}{-4} = -1 + \frac{\sqrt{6}}{2}.$$

5. (a) First we write

$$x^2 + 6x + 8 = (x^2 + 6x) + 8.$$

The coefficient of x is 6. We compute $p = \frac{1}{2} \cdot 6 = 3$ and $q = p^2 = 3^2 = 9$. Now we add q within the brackets and subtract it outside, giving

$$x^2 + 6x - 8 = (x^2 + 6x + 9) + 8 - 9 = (x^2 + 6x + 9) - 1.$$

Finally, the expression in brackets must become $(x+p)^2 = (x+3)^2$. Thus we have

$$x^2 + 6x + 8 = (x+3)^2 - 1.$$

(b) We first factor out the coefficient of x^2:

$$3x^2 - 3x + 2 = 3\left[(x^2 - x) + \frac{2}{3}\right].$$

Now the coefficient of x is -1, so we compute $p = \frac{1}{2}(-1) = -\frac{1}{2}$ and $q = p^2 = \left(-\frac{1}{2}\right)^2 = \frac{1}{4}$. We add q within the brackets and subtract it outside, giving

$$\begin{aligned} 3x^2 - 3x + 2 &= 3\left[\left(x^2 - x + \frac{1}{4}\right) + \frac{2}{3} - \frac{1}{4}\right] \\ &= 3\left[\left(x^2 - x + \frac{1}{4}\right) + \frac{5}{12}\right] \\ &= 3\left[\left(x - \frac{1}{2}\right)^2 + \frac{5}{12}\right] \\ &= 3\left(x - \frac{1}{2}\right)^2 + \frac{5}{4}. \end{aligned}$$

(c) We can rewrite this expression as

$$17 - 5x^2 - 40x = -5x^2 - 40x + 17.$$

Now factor out the coefficient of x^2:

$$17 - 5x^2 - 40x = -5\left[(x^2 + 8x) - \frac{17}{5}\right].$$

The coefficient of x is 8 so $p = \frac{1}{2} \cdot 8 = 4$ and $q = p^2 = 4^2 = 16$. Adding p inside the brackets and subtracting it outside, we obtain

$$\begin{aligned} 17 - 5x^2 - 40x &= -5\left[(x^2 + 8x + 16) - \frac{17}{5} - 16\right] \\ &= -5\left[(x + 4)^2 - \frac{97}{5}\right] \\ &= -5(x + 4)^2 + 97. \end{aligned}$$

6. (a) Since there is no x term, we know immediately that the vertex (and the y-intercept) of the parabola is the point $(0, 1)$.
To find the x-intercepts, we set

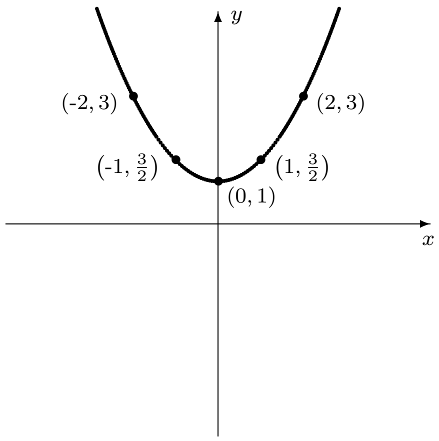
$$\begin{aligned} \frac{1}{2}x^2 + 1 &= 0 \\ x^2 &= -2, \end{aligned}$$

which is impossible. This means that there are no x-intercepts.

We need two points on either side of the vertex. For instance,

$$\begin{aligned} f(1) &= \frac{1}{2}(1^2) + 1 = \frac{3}{2} \\ f(2) &= \frac{1}{2}(2^2) + 1 = 3 \\ f(-1) &= \frac{1}{2}(-1)^2 + 1 = \frac{3}{2} \\ f(-2) &= \frac{1}{2}(-2)^2 + 1 = 3, \end{aligned}$$

so $\left(1, \frac{3}{2}\right)$, $(2, 3)$, $\left(-1, \frac{3}{2}\right)$, and $(-2, 3)$ are all appropriate points. Lastly, we can sketch the graph.



Now we can see that the range is $[1, \infty)$.

(b) We begin by completing the square to write the quadratic equation in standard form:

$$\begin{aligned} y &= -x^2 + 4x - 3 \\ &= -[(x^2 - 4x) + 3] \\ &= -[(x^2 - 4x + 4) + 3 - 4] \\ &= -[(x - 2)^2 - 1] \\ &= -(x - 2)^2 + 1. \end{aligned}$$

This is a parabola with vertex $(2, 1)$.
Now we find any x-intercepts, by solving

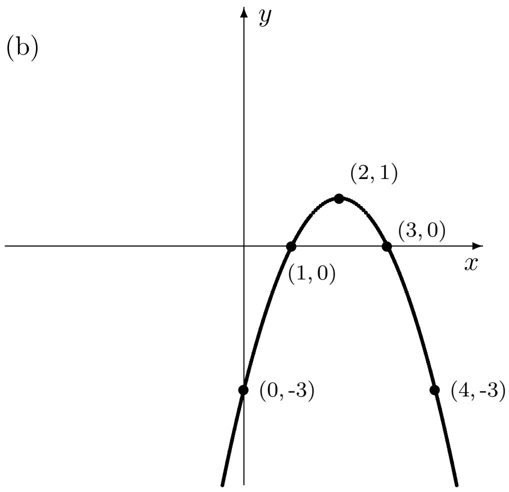
$$\begin{aligned} -x^2 + 4x - 3 &= 0 \\ -(x^2 - 4x + 3) &= 0 \\ -(x - 1)(x - 3) &= 0, \end{aligned}$$

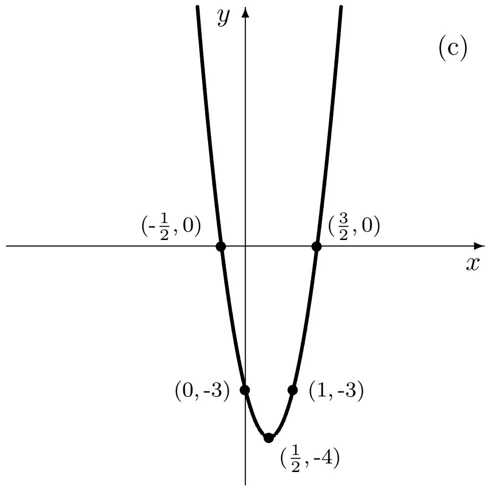
so $x = 1$ or $x = 3$. Therefore the x-intercepts are $(1, 0)$ and $(3, 0)$.

Also, $f(0) = -3$ so the y-intercept is $(0, -3)$. In order to sketch the graph, we simply need one more point to the right of the vertex. For instance, when $x = 4$,

$$f(4) = -(4^2) + 4 \cdot 4 - 3 = -3,$$

so $(4, -3)$ is such a point. Now we can sketch the graph, shown below.
Note that the range is $(-\infty, 1]$.





(c) Again, we must first complete the square:

$$\begin{aligned} y &= 4x^2 - 4x - 3 \\ &= 4\left[(x^2 - x) - \frac{3}{4}\right] \\ &= 4\left[\left(x^2 - x + \frac{1}{4}\right) - \frac{3}{4} - \frac{1}{4}\right] \\ &= 4\left[\left(x - \frac{1}{2}\right)^2 - 1\right] \\ &= 4\left(x - \frac{1}{2}\right)^2 - 4. \end{aligned}$$

Thus the vertex of the parabola is $(\frac{1}{2}, -4)$.
The x-intercepts, if any, are found by setting

$$\begin{aligned} 4x^2 - 4x - 3 &= 0 \\ (2x - 3)(2x + 1) &= 0 \end{aligned}$$

and hence $2x - 3 = 0$ so $x = \frac{3}{2}$ or $2x + 1 = 0$ so $x = -\frac{1}{2}$. Thus the x-intercepts are $(\frac{3}{2}, 0)$ and $(-\frac{1}{2}, 0)$.

We let $x = 0$ so $y = -3$ and hence the y-intercept is $(0, -3)$. We need one more point to the right of the vertex in order to sketch the graph. When $x = 1$,

$$y = 4(1^2) - 4 \cdot 1 - 3 = -3,$$

so $(1, -3)$ is an appropriate point. Now we can sketch the graph, shown above.
Note that the range is $[-4, \infty)$.