SOLUTIONS

1. (a) We have

$$\begin{aligned} 5 - |x-2| &= 0 \\ -|x-2| &= -5 \\ |x-2| &= 5. \end{aligned}$$

We now have two possibilities. If $|x-2| = x-2$ then

$$\begin{aligned} x - 2 &= 5 \\ x &= 7. \end{aligned}$$

On the other hand, if $|x-2| = -(x-2) = -x+2$ then

$$\begin{aligned} -x + 2 &= 5 \\ -x &= 3 \\ x &= -3. \end{aligned}$$

We check both of these values by substituting back into the original equation. When $x = 7$,

$$5 - |x-2| = 5 - |7-2| = 5 - |5| = 5 - 5 = 0,$$

so $x = 7$ is a solution. When $x = -3$,

$$5 - |x-2| = 5 - |-3-2| = 5 - |-5| = 5 - 5 = 0,$$

so $x = -3$ is also a solution. Hence there are two solutions: $x = 7$ and $x = -3$.

(b) If $|3-2x| = 3-2x$ then

$$\begin{aligned} 3 - 2x &= 5 \\ -2x &= 2 \\ x &= -1. \end{aligned}$$

If $|3-2x| = -(3-2x) = -3+2x$ then

$$\begin{aligned} -3 + 2x &= 5 \\ 2x &= 8 \\ x &= 4. \end{aligned}$$

Again, we need to check the validity of these solutions. When $x = -1$,

$$|3 - 2x| = |3 - 2(-1)| = |3 + 2| = |5| = 5,$$

so $x = -1$ is a solution. When $x = 4$,

$$|3 - 2x| = |3 - 2 \cdot 4| = |3 - 8| = |-5| = 5,$$

so $x = 4$ is also a solution. Hence both $x - 1$ and $x = 4$ are solutions of the equation.

(c) If $|x + 4| = x + 4$ then the equation becomes

$$\begin{aligned} x + 4 &= 3 - x \\ 2x &= -1 \\ x &= -\frac{1}{2}. \end{aligned}$$

If $|x + 4| = -(x + 4) = -x - 4$ then we obtain

$$\begin{aligned} -x - 4 &= 3 - x \\ 0 &= 7, \end{aligned}$$

which is not possible. Checking $x = -\frac{1}{2}$ by substitution, we have

$$\left|-\frac{1}{2} + 4\right| = \left|\frac{7}{2}\right| = \frac{7}{2}$$

and

$$3 - x = 3 - \left(-\frac{1}{2}\right) = 3 + \frac{1}{2} = \frac{7}{2},$$

so this is indeed a solution. Therefore $x = -\frac{1}{2}$ is the only solution.

(d) We have

$$\begin{aligned} 3|2 - x| &= 6 - \frac{7x}{3} \\ |2 - x| &= 2 - \frac{7}{9}x. \end{aligned}$$

If $|2 - x| = 2 - x$ then

$$\begin{aligned} 2 - x &= 2 - \frac{7}{9}x \\ -\frac{2}{9}x &= 0 \\ x &= 0. \end{aligned}$$

If $|2-x| = -(2-x) = -2+x$ then

$$\begin{aligned} -2+x &= 2-\frac{7}{9}x \\ \frac{16}{9}x &= 4 \\ x &= \frac{9}{4}. \end{aligned}$$

We check both solutions. When $x=0$,

$$3|2-x| = 3|2-0| = 3|2| = 3\cdot 2 = 6 \quad \text{and} \quad 6-\frac{7x}{3} = 6-\frac{7\cdot 0}{3} = 6-0 = 6,$$

so this is a solution. When $x=\frac{9}{4}$,

$$3\left|2-\frac{9}{4}\right| = 3\left|-\frac{1}{4}\right| = 3\cdot\frac{1}{4} = \frac{3}{4} \quad \text{and} \quad 6-\frac{7x}{3} = 6-\frac{7\cdot\frac{9}{4}}{3} = 6-\frac{21}{4} = \frac{3}{4},$$

so this is also a solution. Thus there are two solutions: $x=0$ and $x=\frac{9}{4}$.

(e) We have

$$\begin{aligned} 3x-\frac{1}{4}|4x-1| &= 0 \\ -\frac{1}{4}|4x-1| &= -3x \\ |4x-1| &= 12x. \end{aligned}$$

If $|4x-1| = 4x-1$ then

$$\begin{aligned} 4x-1 &= 12x \\ -8x &= 1 \\ x &= -\frac{1}{8}. \end{aligned}$$

If $|4x-1| = -(4x-1) = -4x+1$ then

$$\begin{aligned} -4x+1 &= 12x \\ -16x &= -1 \\ x &= \frac{1}{16}. \end{aligned}$$

We verify both possibilities. For $x=-\frac{1}{8}$,

$$3x-\frac{1}{4}|4x-1| = 3\left(-\frac{1}{8}\right)-\frac{1}{4}\left|4\left(-\frac{1}{8}\right)-1\right| = -\frac{3}{8}-\frac{1}{4}\left|-\frac{3}{2}\right| = -\frac{3}{8}-\frac{1}{4}\cdot\frac{3}{2} = -\frac{3}{8}-\frac{3}{8} = -\frac{3}{4},$$

so this is not a solution. For $x = \frac{1}{16}$,

$$3x - \frac{1}{4}|4x-1| = 3\cdot\frac{1}{16} - \frac{1}{4}\left|4\cdot\frac{1}{16} - 1\right| = \frac{3}{16} - \frac{1}{4}\left|-\frac{3}{4}\right| = \frac{3}{16} - \frac{1}{4}\cdot\frac{3}{4} = \frac{3}{16} - \frac{3}{16} = 0,$$

so this is a solution. Hence the only solution is $x = \frac{1}{16}$.

2. (a) First we rewrite the given function as a piecewise function. We solve the inequality

$$\begin{aligned} 1 - \frac{x}{3} &\geq 0 \\ 1 &\geq \frac{1}{3}x \\ 3 &\geq x. \end{aligned}$$

This means that

$$\left|1 - \frac{x}{3}\right| = \begin{cases} 1 - \frac{x}{3} & \text{for } x \leq 3 \\ -1 + \frac{x}{3} & \text{for } x > 3 \end{cases}$$

and so the vertex is located at $x = 3$. At this point,

$$y = \left|1 - \frac{x}{3}\right| = \left|1 - \frac{3}{3}\right| = |1-1| = |0| = 0,$$

so the vertex is the point $(3, 0)$.

This must also be the (only) x-intercept, but to check this we can solve the equation

$$\left|1 - \frac{x}{3}\right| = 0.$$

Either

$$\begin{aligned} 1 - \frac{x}{3} &= 0 \\ -\frac{x}{3} &= -1 \\ x &= 3 \end{aligned}$$

or

$$\begin{aligned} -1 + \frac{x}{3} &= 0 \\ \frac{x}{3} &= 1 \\ x &= 3. \end{aligned}$$

Either way, we see that $(3, 0)$ is indeed the only x-intercept.

To find the y-intercept, we set $x = 0$ and obtain

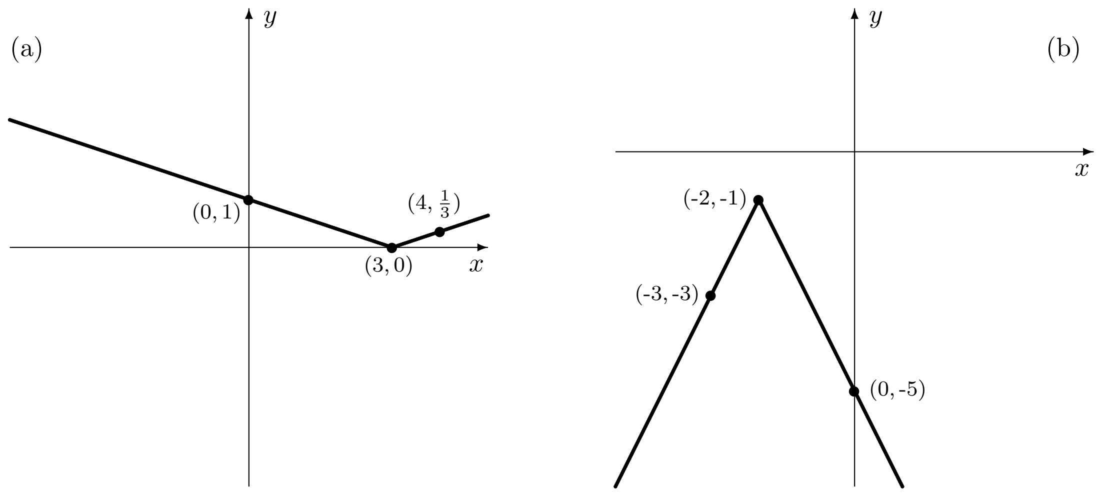
$$y = \left|1 - \frac{0}{3}\right| = |1-0| = |1| = 1,$$

so $(0,1)$ is the y-intercept.

This is a point to the left of the x-intercept (since $0 < 3$) so to sketch the graph we also need a point to the right. If we choose $x = 4$ then

$$y = \left|1 - \frac{4}{3}\right| = \left|-\frac{1}{3}\right| = \frac{1}{3},$$

so $\left(4, \frac{1}{3}\right)$ is such a point. (Another good choice would be $x = 6$, since then we won't have to deal with fractions. In that case, we would obtain the point $(6,1)$.) Now we can sketch the graph, below.



(b) We set

$$\begin{aligned} x + 2 &\geq 0 \\ x &\geq -2. \end{aligned}$$

This means that

$$|x+2| = \begin{cases} x+2 & \text{for } x \geq -2 \\ -x-2 & \text{for } x < -2 \end{cases}$$

and so

$$-2|x+2| = \begin{cases} -2x-4 & \text{for } x \geq -2 \\ 2x+4 & \text{for } x < -2 \end{cases}$$

and finally

$$-2|x+2| - 1 = \begin{cases} -2x-5 & \text{for } x \geq -2 \\ 2x+3 & \text{for } x < -2. \end{cases}$$

The vertex is at $x = -2$, and

$$f(-2) = -2|-2+2| - 1 = -2|0| - 1 = 0 - 1 = -1$$

so this is the point $(-2,-1)$.

The x-intercepts can be found by setting

$$\begin{aligned} -2|x+2|-1 &= 0 \\ -2|x+2| &= 1 \\ |x+2| &= -\frac{1}{2}, \end{aligned}$$

which is impossible since an absolute value is never negative. Thus there are no solutions to this equations, and therefore there are no x-intercepts.
For the y-intercept, we calculate

$$f(0) = -2|0+2|-1 = -2|2|-1 = -2\cdot 2-1 = -5$$

so $(0,-5)$ is the y-intercept.
We need one point to the left of the vertex, such as at $x=-3$. Here,

$$f(-3) = -2|-3+2|-1 = -2|-1|-1 = -2\cdot 1-1 = -3$$

and so $(-3,-3)$ is a point on the graph. Now we can sketch the graph, as found above.