MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 2.2 (PART ONE) **Math 1090** FALL 2009

SOLUTIONS

1. (a) $|7-3| = |4| = 4$

   (b) $|3-7| = |-4| = 4$

   (c) $|-3-7| = |-10| = 10$

   (d) $|8| - |-1| = 8 - 1 = 7$

   (e) $|-2 \cdot 4| = |-8| = 8$

   (f) $-2 \cdot |4| = -2 \cdot 4 = -8$

   (g) $\dfrac{5}{|-5|} = \dfrac{5}{5} = 1$

   (h) $\dfrac{|-3+3|}{3} = \dfrac{|0|}{3} = \dfrac{0}{3} = 0$

   (i) $\dfrac{|-3|+|3|}{3} = \dfrac{3+3}{3} = \dfrac{6}{3} = 2$

2. (a) $f(2) = |6 - 3 \cdot 2| - 2 = |6-6| - 2 = |0| - 2 = 0 - 2 = -2$

   (b) $f\left(\dfrac{11}{3}\right) = \left|6 - 3 \cdot \dfrac{11}{3}\right| - 2 = |6-11| - 2 = |-5| - 2 = 5 - 2 = 3$

   (c) $f(-1) = |6 - 3(-1)| - 2 = |6+3| - 2 = |9| - 2 = 9 - 2 = 7$

3. (a) $g(4) = \left|3 \cdot 4 - \dfrac{5}{2} \cdot |4|\right| = \left|12 - \dfrac{5}{2} \cdot 4\right| = |12 - 10| = |2| = 2$

   (b) $g(-4) = \left|3(-4) - \dfrac{5}{2} \cdot |-4|\right| = \left|-12 - \dfrac{5}{2} \cdot 4\right| = |-12 - 10| = |-22| = 22$

   (c) $g(-1) = \left|3(-1) - \dfrac{5}{2} \cdot |-1|\right| = \left|-3 - \dfrac{5}{2} \cdot 1\right| = \left|-3 - \dfrac{5}{2}\right| = \left|-\dfrac{11}{2}\right| = \dfrac{11}{2}$

4. (a) Note that

$$f(x) = \left|\frac{1}{3}x\right| = \frac{1}{3}|x|.$$

   So, using the definition of $|x|$,

$$f(x) = \frac{1}{3}\begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases}$$

$$= \begin{cases} \frac{1}{3}x & \text{for } x \geq 0 \\ -\frac{1}{3}x & \text{for } x < 0. \end{cases}$$

(b) First we simplify:

$$y = \frac{|4x| + 6}{2} = \frac{4|x| + 6}{2} = 2|x| + 3$$

so

$$2|x| = 2\begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases} = \begin{cases} 2x & \text{for } x \geq 0 \\ -2x & \text{for } x < 0 \end{cases}$$

and therefore

$$y = 2|x| + 3 = \begin{cases} 2x + 3 & \text{for } x \geq 0 \\ -2x + 3 & \text{for } x < 0. \end{cases}$$

(c) We can write

$$\begin{aligned} f(x) &= 2 - \begin{cases} x & \text{for } x \geq 0 \\ -x & \text{for } x < 0 \end{cases} \\ &= \begin{cases} 2 - x & \text{for } x \geq 0 \\ 2 + x & \text{for } x < 0. \end{cases} \end{aligned}$$

5. (a) If we divide both sides by -4 then, remembering to change the direction of the inequality, we have

$$\begin{aligned} -4x &> 9 \\ x &< -\frac{9}{4} \end{aligned}$$

so the solution is $\left(-\infty, -\frac{9}{4}\right)$. Alternatively, you might prefer to begin by rearranging the given inequality:

$$\begin{aligned} -4x &> 9 \\ -9 &> 4x \\ -\frac{9}{4} &> x. \end{aligned}$$

Of course, the solution is the same either way.

(b) We bring the terms involving x to the righthand side to ensure that the coefficient of x is positive:

$$\begin{aligned} 4 - 3x &\leq x + 8 \\ -4 &\leq 4x \\ -1 &\leq x, \end{aligned}$$

so the the solution is $[-1, \infty)$.

(c) We have

$$\begin{aligned} 2 &\geq \frac{1}{3}(6-x) \\ 2 &\geq 2 - \frac{1}{3}x \\ \frac{1}{3}x &\geq 0 \\ x &\geq 0. \end{aligned}$$

The solution is $[0, \infty)$.