

Section 1.4: Absolute Value

Def'n: The absolute value (or modulus) of a number x is

$$|x| = \begin{cases} x, & \text{for } x \geq 0 \\ -x, & \text{for } x < 0 \end{cases}$$

eg $|\frac{1}{2}| = \frac{1}{2}$ because $\frac{1}{2} \geq 0$

$|0| = 0$ because $0 \geq 0$

$|-4| = -(-4) = 4$ because $-4 < 0$

Properties of the absolute value

① For any real number x , $|x| \geq 0$

② $|-x| = |x|$

③ $|xy| = |x| \cdot |y|$

④ $|\frac{x}{y}| = \frac{|x|}{|y|}$ for $y \neq 0$

The absolute value bars are treated as brackets for mathematical operations.

eg $|4-6| = |-2| = 2$

$4 - |-6| = 4 - 6 = -2$

If we are given information about any variables present, we can often simplify expressions involving absolute values.

eg Suppose $x < 0$. Simplify $7 - |3x|$.

$$\text{We have } 7 - |3x| = 7 - (|3| \cdot |x|)$$

$$= 7 - 3|x|$$

$$= 7 - 3 \cdot (-x)$$

$$\boxed{= 7 + 3x}$$

eg Suppose $x \geq 4$. Simplify $|x-4| + |4-x|$.

We have

$$|x-4| = \begin{cases} x-4, & \text{for } x-4 \geq 0 \text{ so } x \geq 4 \\ -(x-4), & \text{for } x-4 < 0 \text{ so } x < 4 \end{cases}$$

$$\text{and } |4-x| = \begin{cases} 4-x, & \text{for } 4-x \geq 0 \text{ so } x \leq 4 \\ -(4-x), & \text{for } 4-x < 0 \text{ so } x > 4 \end{cases}$$

$$\text{Therefore } |x-4| + |4-x|$$

$$= (x-4) + [-(4-x)]$$

$$= x-4-4+x$$

$$\boxed{= 2x-8}$$