For practice only. Not to be submitted.

1. Write each of the following sums using sigma notation (with $i = 1$ as the lower bound of summation).

   (a) $\dfrac{2}{5} + \dfrac{4}{10} + \dfrac{8}{15} + \dfrac{16}{20} + \cdots + \dfrac{2^n}{5n}$

   (b) $y^2 + 8y^2 + 27y^2 + 64y^2 + \cdots + n^3 y^2$

2. Use the summation formulas to evaluate each of the following sums.

   (a) $\displaystyle\sum_{i=1}^{n}(4i + 3)$

   (b) $\displaystyle\sum_{i=1}^{n}(i^3 - 6i)$

   (c) $\displaystyle\sum_{i=1}^{n}(3i + 1)^2$

3. Consider the region R which lies between the graph of $f(x) = 2 - x$ and the x-axis, on the interval $[-1, 1]$.

   (a) Determine the width Δx of each subinterval in a regular partition of $[-1, 1]$.

   (b) Give an expression for the right endpoint x_i of the ith subinterval. Use this to find formulas for the point m_i and M_i at which $f(x)$ attains its minimum and maximum values on the ith subinterval.

   (c) Determine $f(m_i)$ and $f(M_i)$.

   (d) Find formulas for the lower sum $s(n)$ and the upper sum $S(n)$.

   (e) Estimate the area A of the region R by finding the lower and upper sums with $n = 5$ subintervals. Give your answers to one decimal place.

   (f) Estimate the area A of the region R by finding the lower and upper sums with $n = 500$ subintervals. Give your answers to two decimal places.

   (g) Find the true value of A by taking the limit of the formulas you derived in part (d) as $n \to \infty$, and showing that these limits are equal.

4. Use the limit of a Riemann sum to determine the area A of the region which lies below the curve $f(x) = 3 + 3x - x^2$, above the x-axis, to the left of $x = -1$ and to the right of $x = 3$.