MEMORIAL UNIVERSITY OF NEWFOUNDLAND DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 5	Mathematics 1001	WINTER	2025

SOLUTIONS

[4] 1. We write

$$f(x) = \int_{x}^{0} \sqrt{1+t^{4}} \, dt + \int_{0}^{5x} \sqrt{1+t^{4}} \, dt = -\int_{0}^{x} \sqrt{1+t^{4}} \, dt + \int_{0}^{5x} \sqrt{1+t^{4}} \, dt.$$

Now

$$f'(x) = -\sqrt{1+x^4} + \sqrt{1+(5x)^4} \cdot 5 = -\sqrt{1+x^4} + 5\sqrt{1+625x^4}.$$

[5] 2. (a) Let $u = \sin(\theta)$ so $du = \cos(\theta) d\theta$. When $\theta = \frac{\pi}{6}$, $u = \sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$. When $\theta = \frac{\pi}{4}$, $u = \sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$. The integral becomes

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \sin^3(\theta) \cos(\theta) \, d\theta = \int_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}} u^3 \, du$$
$$= \frac{1}{4} \left[u^4 \right]_{\frac{1}{2}}^{\frac{\sqrt{2}}{2}}$$
$$= \frac{1}{4} \left[\frac{1}{4} - \frac{1}{16} \right]$$
$$= \frac{3}{64}.$$

[4] (b) We use integration by parts. Let $w = \ln(x)$ so $dw = \frac{1}{x} dx$. Let $dv = \frac{1}{x^3} dx$ so $v = -\frac{1}{2x^2}$. Then

$$\int_{1}^{e} \frac{\ln(x)}{x^{3}} dx = \left[-\frac{\ln(x)}{2x^{2}} \right]_{1}^{e} + \frac{1}{2} \int_{1}^{e} \frac{1}{x^{3}} dx$$
$$= \left[-\frac{\ln(x)}{2x^{2}} - \frac{1}{4x^{2}} \right]_{1}^{e}$$
$$= -\frac{3}{4e^{2}} + \frac{1}{4}.$$

[4] (c) Observe that |x+2| = x+2 when $x+2 \ge 0$, so $x \ge -2$. On the other hand, |x+2| = -(x+2) when x+2 < 0, so x < -2. This means that we can write

$$|x+2| = \begin{cases} x+2, & \text{for } x \ge -2\\ -(x+2), & \text{for } x < -2 \end{cases}$$

and the integral becomes

$$\int_{-5}^{5} |x+2| \, dx = \int_{-5}^{-2} |x+2| \, dx + \int_{-2}^{5} |x+2| \, dx$$
$$= -\int_{-5}^{-2} (x+2) \, dx + \int_{-2}^{5} (x+2) \, dx$$
$$= -\left[\frac{1}{2}x^2 + 2x\right]_{-5}^{-2} + \left[\frac{1}{2}x^2 + 2x\right]_{-2}^{5}$$
$$= -\left[-2 - \frac{5}{2}\right] + \left[\frac{45}{2} - (-2)\right]$$
$$= 29.$$

[3] 3. The area under the curve is given by

$$A = \int_0^5 \frac{1}{x^2 + 25} dx$$

= $\frac{1}{5} \left[\arctan\left(\frac{x}{5}\right) \right]_0^5$
= $\frac{1}{5} \left[\arctan(1) - \arctan(0) \right]$
= $\frac{1}{5} \left[\frac{\pi}{4} - 0 \right]$
= $\frac{\pi}{20}$.