MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 5 | **MATHEMATICS 1001** | WINTER 2023

SOLUTIONS

[4] 1. We write

$$\int_{-x}^{\sqrt{x}} \cos(t^2)\,dt = \int_{-x}^{0} \cos(t^2)\,dt + \int_{0}^{\sqrt{x}} \cos(t^2)\,dt = -\int_{0}^{-x} \cos(t^2)\,dt + \int_{0}^{\sqrt{x}} \cos(t^2)\,dt.$$

Then

$$f'(x) = -\cos((-x)^2)\cdot[-x]' + \cos\left(\left(\sqrt{x}\right)^2\right)\cdot\left[\sqrt{x}\right]' = \cos(x^2) + \frac{\cos(x)}{2\sqrt{x}}.$$

[4] 2. (a) We use integration by parts with $w = \ln(x)$ so $dw = \frac{1}{x}\,dx$, and $dv = \sqrt{x}\,dx$ so $v = \frac{2}{3}x^{\frac{3}{2}}$. Thus

$$\begin{aligned}
\int_1^e \sqrt{x}\ln(x)\,dx &= \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x)\right]_1^e - \int_1^e \frac{2}{3}x^{\frac{3}{2}}\cdot\frac{1}{x}\,dx \\
&= \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x)\right]_1^e - \frac{2}{3}\int_1^e x^{\frac{1}{2}}\,dx \\
&= \left[\frac{2}{3}x^{\frac{3}{2}}\ln(x) - \frac{4}{9}x^{\frac{3}{2}}\right]_1^e \\
&= \left[\frac{2}{3}e^{\frac{3}{2}}\ln(e) - \frac{4}{9}e^{\frac{3}{2}}\right] - \left[\frac{2}{3}\ln(1) - \frac{4}{9}\right] \\
&= \frac{2}{9}e^{\frac{3}{2}} + \frac{4}{9}.
\end{aligned}$$

[3] (b) Let $u = \cos(2t)$ so $du = -2\sin(2t)\,dt$ and $-\frac{1}{2}\,du = \sin(2t)\,dt$. When $t = 0$, $u = \cos(0) = 1$. When $t = \pi$, $u = \cos(2\pi) = 1$. Thus the integral becomes

$$\int_0^{\pi} \sin(2t)\cos(2t)\sin(\cos(2t))\,dt = -\frac{1}{2}\int_1^1 u\sin(u)\,du.$$

We could now evaluate this integral using integration by parts, but it is much easier to simply observe that the bounds of integration are now the same, and therefore we can immediately conclude that

$$\int_0^{\pi} \sin(2t)\cos(2t)\sin(\cos(2t))\,dt = 0.$$

[3] 3. We can write

$$\begin{aligned}\int_{-1}^{9} f(x)\,dx &= \int_{-1}^{1} f(x)\,dx + \int_{1}^{9} f(x)\,dx\\ &= \int_{-1}^{1} (-4)\,dx + \int_{1}^{9} \left(3\sqrt{x}-7\right)\,dx\\ &= \Big[-4x\Big]_{-1}^{1} + \Big[2x^{\frac{3}{2}} - 7x\Big]_{1}^{9}\\ &= -8 + (-4)\\ &= -12.\end{aligned}$$

[6] 4. Since the y-axis is the line $x = 0$, we have

$$A = \int_{-\frac{3}{2}}^{0} \frac{1}{2x^2+6x+9}\,dx.$$

We can complete the square:

$$\begin{aligned}2x^2+6x+9 &= 2\left[x^2+3x+\frac{9}{2}\right]\\ &= 2\left[\left(x^2+3x+\frac{9}{4}\right)+\frac{9}{2}-\frac{9}{4}\right]\\ &= 2\left[\left(x+\frac{3}{2}\right)^2+\frac{9}{4}\right]\\ &= 2\left(x+\frac{3}{2}\right)^2+\frac{9}{2}.\end{aligned}$$

(In this case, we elect not to multiply the factor of 2 back into the expression in brackets, to avoid introducing a radical.) So now we have

$$A = \int_{-\frac{3}{2}}^{0} \frac{1}{2\left(x+\frac{3}{2}\right)^2+\frac{9}{2}}\,dx = \frac{1}{2}\int_{-\frac{3}{2}}^{0} \frac{1}{\left(x+\frac{3}{2}\right)^2+\frac{9}{4}}\,dx.$$

Now we let $u = x + \frac{3}{2}$ so $du = dx$. When $x = -\frac{3}{2}$, $u = 0$. When $x = 0$, $u = \frac{3}{2}$. So the integral becomes

$$
\begin{aligned}
A &= \frac{1}{2}\int_0^{\frac{3}{2}} \frac{1}{u^2+\frac{9}{4}}\,du \\
&= \frac{1}{2}\left[\frac{1}{\frac{3}{2}}\arctan\left(\frac{u}{\frac{3}{2}}\right)\right]_0^{\frac{3}{2}} \\
&= \frac{1}{3}\left[\arctan\left(\frac{2u}{3}\right)\right]_0^{\frac{3}{2}} \\
&= \frac{1}{3}[\arctan(1)-\arctan(0)] \\
&= \frac{1}{3}\left[\frac{\pi}{4}-0\right] \\
&= \frac{\pi}{12}.
\end{aligned}
$$