MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

ASSIGNMENT 4 **MATHEMATICS 1001** WINTER 2023

SOLUTIONS

[5] 1. (a) We form a regular partition where

$$\Delta x = \frac{2-(-1)}{n} = \frac{3}{n}$$

and choose the sample point

$$x_i^* = x_i = -1 + \frac{3i}{n}.$$

Thus

$$\begin{aligned} f(x_i) &= f\left(-1 + \frac{3i}{n}\right) \\ &= \left(-1 + \frac{3i}{n}\right)^3 + 5 \\ &= \frac{27i^3}{n^3} - \frac{27i^2}{n^2} + \frac{9i}{n} + 4. \end{aligned}$$

Now we have

$$\begin{aligned} A &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{27i^3}{n^3} - \frac{27i^2}{n^2} + \frac{9i}{n} + 4\right] \cdot \frac{3}{n} \\ &= \lim_{n\to\infty} \left[\frac{81}{n^4}\sum_{i=1}^{n} i^3 - \frac{81}{n^3}\sum_{i=1}^{n} i^2 + \frac{27}{n^2}\sum_{i=1}^{n} i + \frac{12}{n}\sum_{i=1}^{n} 1\right] \\ &= \lim_{n\to\infty} \left[\frac{81}{n^4} \cdot \frac{n^2(n+1)^2}{4} - \frac{81}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{27}{n^2} \cdot \frac{n(n+1)}{2} + \frac{12}{n} \cdot n\right] \\ &= \frac{81}{4} - 27 + \frac{27}{2} + 12 \\ &= \frac{75}{4}. \end{aligned}$$

[5] (b) We form a regular partition where

$$\Delta x = \frac{5-0}{n} = \frac{5}{n}$$

and choose the sample point

$$x_i^* = x_i = 0 + \frac{5i}{n} = \frac{5i}{n}.$$

Thus

$$\begin{aligned} f(x_i) &= f\left(\frac{5i}{n}\right) \\ &= \left(3 \cdot \frac{5i}{n} - 1\right)^2 \\ &= \frac{225i^2}{n^2} - \frac{30i}{n} + 1. \end{aligned}$$

Now we have

$$\begin{aligned} A &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{225i^2}{n^2} - \frac{30i}{n} + 1\right] \cdot \frac{5}{n} \\ &= \lim_{n\to\infty} \left[\frac{1125}{n^3} \sum_{i=1}^{n} i^2 - \frac{150}{n^2} \sum_{i=1}^{n} i + \frac{5}{n} \sum_{i=1}^{n} 1\right] \\ &= \lim_{n\to\infty} \left[\frac{1125}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{150}{n^2} \cdot \frac{n(n+1)}{2} + \frac{5}{n} \cdot n\right] \\ &= 375 - 75 + 5 \\ &= 305. \end{aligned}$$

[5] 2. Since the y-axis is the line $x = 0$, this region lies on the interval $[0, b]$. Thus we form a regular partition where

$$\Delta x = \frac{b - 0}{n} = \frac{b}{n}$$

and choose the sample point

$$x_i^* = x_i = 0 + \frac{bi}{n} = \frac{bi}{n}.$$

So then

$$f(x_i) = f\left(\frac{bi}{n}\right) = \left(\frac{bi}{n}\right)^2 = \frac{b^2 i^2}{n^2}.$$

Now we have

$$
\begin{aligned}
A &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\
&= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{b^2 i^2}{n^2} \cdot \frac{b}{n} \\
&= \lim_{n\to\infty} \left[\frac{b^3}{n^3} \sum_{i=1}^{n} i^2 \right] \\
&= \lim_{n\to\infty} \left[\frac{b^3}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} \right] \\
&= b^3 \cdot \frac{1}{3} \\
&= \frac{b^3}{3}
\end{aligned}
$$

as desired.

[5] 3. We form a regular partition of $[2,3]$ into n subintervals of length

$$\Delta x = \frac{3-2}{n} = \frac{1}{n}.$$

As the sample point, we choose

$$x_i = a_i = 2 + i \cdot \frac{1}{n} = 2 + \frac{i}{n}$$

so

$$f(x_i) = f\left(2 + \frac{i}{n}\right) = \frac{4i^3}{n^3} + \frac{27i^2}{n^2} + \frac{60i}{n} + 44.$$

Thus

$$\int_2^3 x^2(4x+3)\,dx$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\frac{4i^3}{n^3} + \frac{27i^2}{n^2} + \frac{60i}{n} + 44\right] \cdot \frac{1}{n}$$

$$= \lim_{n\to\infty} \left[\frac{4}{n^4} \sum_{i=1}^{n} i^3 + \frac{27}{n^3} \sum_{i=1}^{n} i^2 + \frac{60}{n^2} \sum i + \frac{44}{n} \sum_{i=1}^{n} 1\right]$$

$$= \lim_{n\to\infty} \left[\frac{4}{n^4} \cdot \frac{n^2(n+1)^2}{4} + \frac{27}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{60}{n^2} \cdot \frac{n(n+1)}{2} + \frac{44}{n} \cdot n\right]$$

$$= 1 + 9 + 30 + 44$$

$$= 84.$$