

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 4.6

Math 1001 Worksheet

WINTER 2025

SOLUTIONS

1. (a) The graph of the region can be found in Figure 1. Since the x -axis (the line $y = 0$) is one of the boundary curves, this is just a disc method problem with $R(x) = \sqrt{x-2}$. Thus

$$\begin{aligned} V &= \pi \int_2^6 (\sqrt{x-2})^2 dx \\ &= \pi \int_2^6 (x-2) dx \\ &= \pi \left[\frac{1}{2}x^2 - 2x \right]_2^6 \\ &= \pi[(18-12) - (2-4)] \\ &= 8\pi. \end{aligned}$$

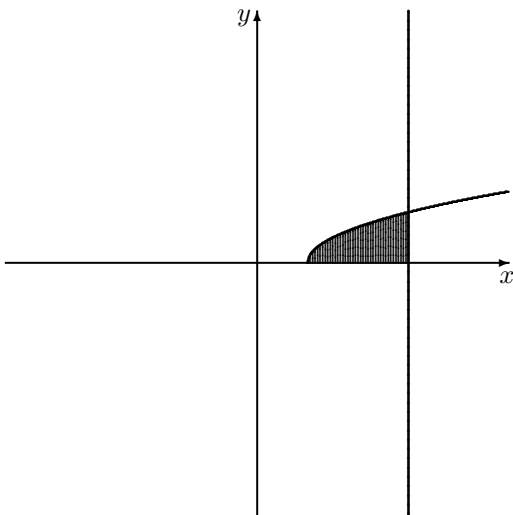


Figure 1: Question 1(a)

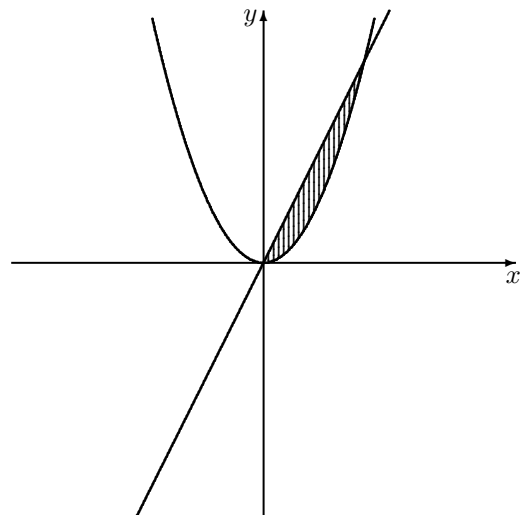


Figure 2: Question 1(b)

- (b) The graph of the region can be found in Figure 2. The boundary curve $y = 2x$ is farther from the axis of revolution than the boundary curve $y = x^2$. Hence the outer radius is $R(x) = 2x$ and the inner radius is $r(x) = x^2$. Also, we find the endpoints of the interval of integration by setting

$$x^2 = 2x \implies x^2 - 2x = x(x-2) = 0$$

giving $x = 0$ and $x = 2$. Thus

$$\begin{aligned}
 V &= \pi \int_0^2 ([2x]^2 - [x^2]^2) dx \\
 &= \pi \int_0^2 (4x^2 - x^4) dx \\
 &= \pi \left[\frac{4}{3}x^3 - \frac{1}{5}x^5 \right]_0^2 \\
 &= \pi \left[\frac{32}{3} - \frac{32}{5} - 0 + 0 \right] \\
 &= \frac{64\pi}{15}.
 \end{aligned}$$

- (c) The graph of the region can be found in Figure 3. The boundary curve closest to the axis is always $y = 2$ — so that $r(x) \equiv 2$ — the farther boundary curve is $y = \frac{12}{x}$ for some values of x , and $y = 3x$ for others. To determine the point where this changes, we set

$$\frac{12}{x} = 3x \implies x^2 = 4 \implies x = \pm 2,$$

of which only $x = 2$ is relevant to the indicated region. Also, we need the left and right endpoints of the entire interval of integration. The left endpoint is the intersection of $y = 2$ and $y = 3x$, namely $x = \frac{2}{3}$. The right endpoint is the intersection of $y = 2$ and $y = \frac{12}{x}$, which is $x = 6$. Hence on $[\frac{2}{3}, 2]$ the outer radius is $R(x) = 3x$ while on $[2, 6]$ the outer radius is $R(x) = \frac{12}{x}$. Thus we calculate

$$\begin{aligned}
 V &= \pi \int_{\frac{2}{3}}^2 ([3x]^2 - [2]^2) dx + \pi \int_2^6 \left(\left[\frac{12}{x} \right]^2 - [2]^2 \right) dx \\
 &= \pi \int_{\frac{2}{3}}^2 (9x^2 - 4) dx + \pi \int_2^6 \left(\frac{144}{x^2} - 4 \right) dx \\
 &= \pi \left[\left[3x^3 - 4x \right] \right]_{\frac{2}{3}}^2 + \pi \left[-\frac{144}{x} - 4x \right]_2^6 \\
 &= \pi \left[(24 - 8) - \left(\frac{8}{9} - \frac{8}{3} \right) \right] + \pi [(-24 - 24) - (-72 - 8)] \\
 &= \frac{160\pi}{9} + 32\pi \\
 &= \frac{448\pi}{9}.
 \end{aligned}$$

2. The plane shape which will generate the desired cone by revolving it around the x -axis is a right triangle with base b and height a . We can form such a plane shape by considering the

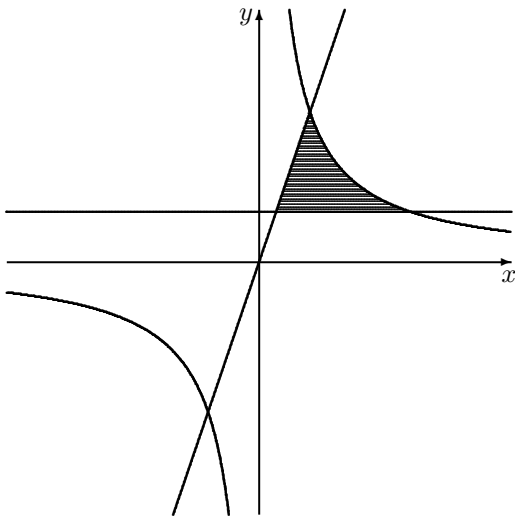


Figure 3: Question 1(c)

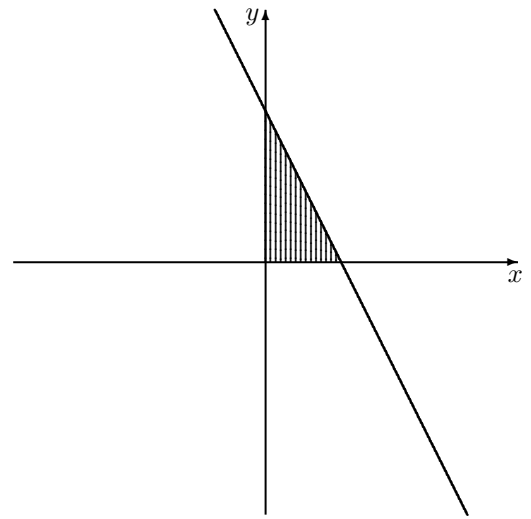


Figure 4: Question 2

region bounded by the curves $y = -\frac{a}{b}x + a$, $y = 0$ and $x = 0$, as depicted in Figure 4. The result is a disc method problem with outer radius $R(x) = -\frac{a}{b}x + a$, so

$$\begin{aligned}
 V &= \pi \int_0^b \left[-\frac{a}{b}x + a \right]^2 dx \\
 &= \pi \int_0^b \left[\frac{a^2}{b^2}x^2 - \frac{2a^2}{b}x + a^2 \right] dx \\
 &= \pi \left[\frac{a^2}{3b^2}x^3 - \frac{a^2}{b}x^2 + a^2x \right]_0^b \\
 &= \frac{\pi}{3}a^2b.
 \end{aligned}$$