MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SOLUTIONS

1. (a) First we solve for the points of intersection, setting

$$\begin{aligned}\sqrt{2x+4} &= x+2 \\ 2x+4 &= (x+2)^2 = x^2+4x+4 \\ x^2+2x = x(x+2) &= 0\end{aligned}$$

so $x = 0$ and $x = -2$. Note that $\sqrt{2x+4} \geq x+2$, as can be shown algebraically or graphically (see Figure 1), so we compute

$$A = \int_{-2}^{0} \left[\sqrt{2x+4} - (x+2)\right]\, dx = \left[\frac{1}{3}(2x+4)^{\frac{3}{2}} - \frac{1}{2}x^2 - 2x\right]_{-2}^{0} = \frac{2}{3}.$$

(b) From part (a), we see that when $x = 0$, $y = 2$, and when $x = -2$, $y = 0$, so these are the points of intersection. The line can be written $x = y - 2$ and the square root function can be written $x = \frac{1}{2}y^2 - 2$. Note that $y - 2 \geq \frac{1}{2}y^2 - 2$, so

$$A = \int_{0}^{2} \left[(y-2) - \left(\frac{1}{2}y^2 - 2\right)\right] dy = \int_{0}^{2} \left[-\frac{1}{2}y^2 + y\right] dy = \left[-\frac{1}{6}y^3 + \frac{1}{2}y^2\right]_{0}^{2} = \frac{2}{3}.$$

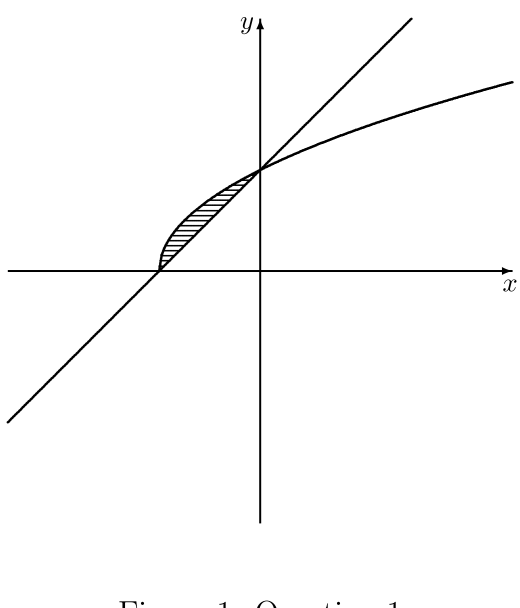

Figure 1: Question 1

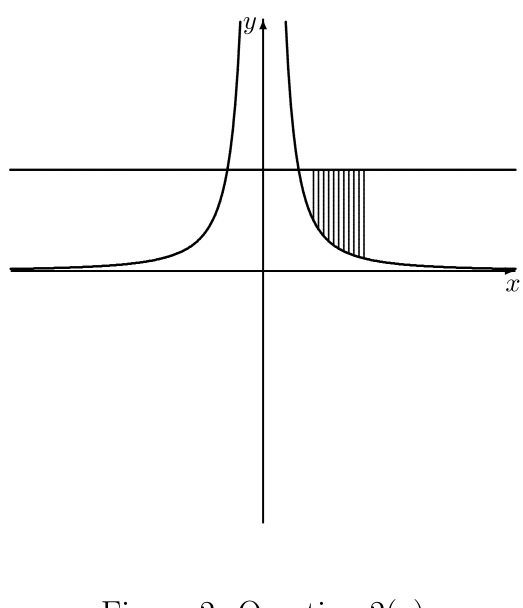

Figure 2: Question 2(a)

2. (a) It should be clear to you that $2 \geq \frac{1}{x^2}$ for all x in the interval $[1, 2]$. Alternatively, you can sketch the graph (see Figure 2). Then

$$A = \int_1^2 \left(2 - \frac{1}{x^2}\right) dx = \left[2x + \frac{1}{x}\right]_1^2 = \left[4 + \frac{1}{2}\right] - [2 + 1] = \frac{3}{2}.$$

(b) We solve for the points of intersection:

$$x^2 + 3x = x + 3 \implies x^2 + 2x - 3 = 0 \implies (x + 3)(x - 1) = 0$$

so $x = 1$ and $x = -3$. Note that $x+3 \geq x^2+3x$ on $[-3, 1]$, as can be shown algebraically or graphically (see Figure 3). Then

$$\begin{aligned} A &= \int_{-3}^{1} [(x + 3) - (x^2 + 3x)]\, dx \\ &= \int_{-3}^{1} [-x^2 - 2x + 3]\, dx \\ &= \left[-\frac{1}{3}x^3 - x^2 + 3x\right]_{-3}^{1} \\ &= \left[-\frac{1}{3} - 1 + 3\right] - [9 - 9 - 9] \\ &= \frac{32}{3}. \end{aligned}$$

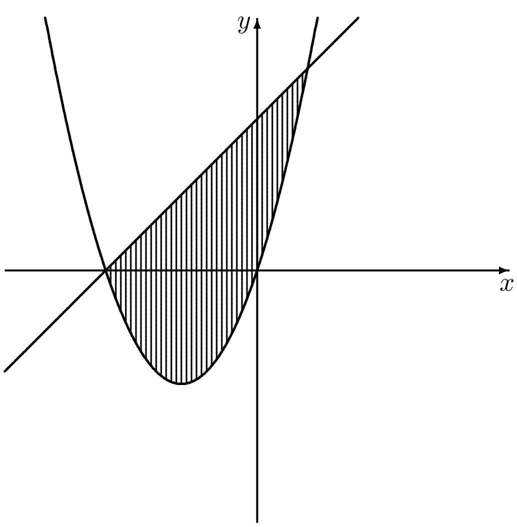

Figure 3: Question 2(b)

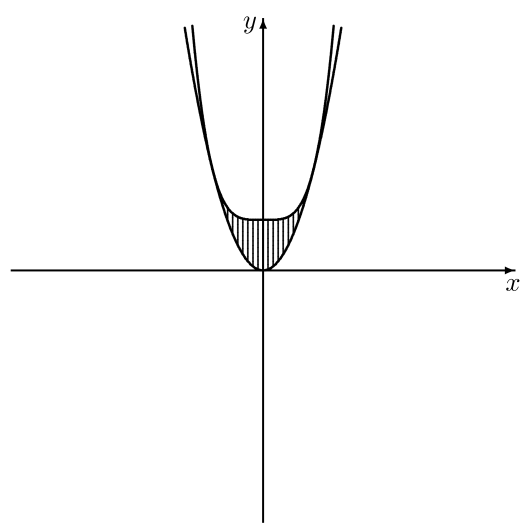

Figure 4: Question 2(c)

(c) We solve for the points of intersection:

$$x^4 + 1 = 2x^2 \quad \Longrightarrow \quad x^4 - 2x^2 + 1 = 0 \quad \Longrightarrow \quad (x^2-1)^2 = 0 \quad \Longrightarrow \quad x^2 = 1$$

so $x = \pm 1$. Note that $x^4 + 1 \geq 2x^2$ on $[-1, 1]$, as can be shown algebraically or graphically (see Figure 4). So

$$\begin{aligned} A &= \int_{-1}^{1} [(x^4+1) - 2x^2]\, dx \\ &= \left[\frac{x^5}{5} + x - \frac{2}{3}x^3\right]_{-1}^{1} \\ &= \left[\frac{1}{5} + 1 - \frac{2}{3}\right] - \left[-\frac{1}{5} - 1 + \frac{2}{3}\right] \\ &= \frac{16}{15}. \end{aligned}$$

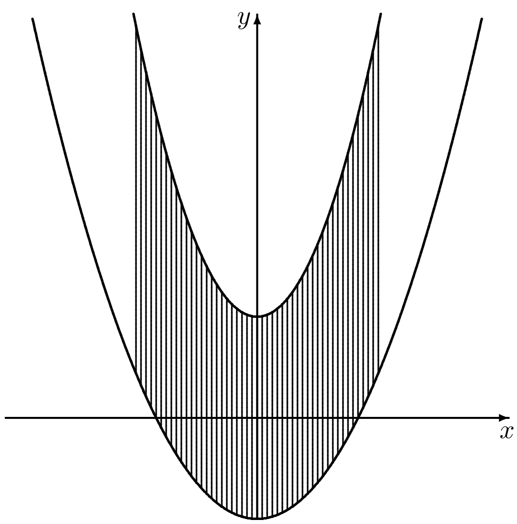

Figure 5: Question 2(d)

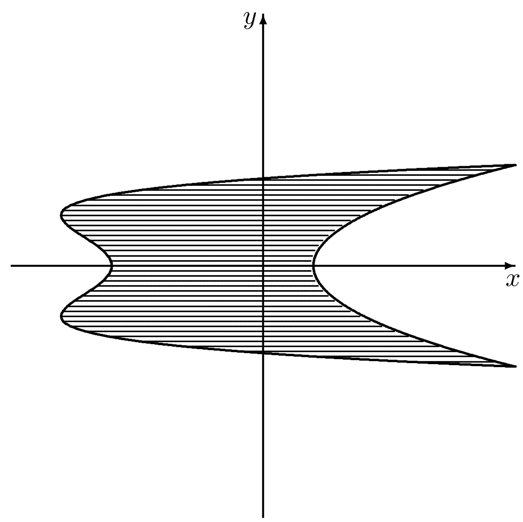

Figure 6: Question 2(e)

(d) It should be clear that $x^2 + 2 \geq \frac{1}{2}x^2 - 2$. therwise, the graph can be found in Figure 5. Thus

$$\begin{aligned} A &= \int_{-3}^{3} \left[(x^2+2) - \left(\frac{1}{2}x^2 - 2\right)\right] dx \\ &= \int_{-3}^{3} \left[\frac{1}{2}x^2 + 4\right] dx \\ &= \left[\frac{1}{6}x^3 + 4x\right]_{-3}^{3} \\ &= 33. \end{aligned}$$

(e) It is very difficult to write $x = y^4 - 2y^2 - 3$ as a function of x, so we will work in terms of functions of y. First we determine the points of intersection, setting

$$y^2 + 1 = y^4 - 2y^2 - 3 \quad \Longrightarrow \quad y^4 - 3y^2 - 4 = (y^2 - 4)(y^2 + 1) = (y + 2)(y - 2)(y^2 + 1) = 0$$

so $y = 2$ and $y = -2$. Note that $y^2 + 1 \geq y^4 - 2y^2 - 3$ on $[-2, 2]$, as can be shown algebraically or graphically (see Figure 6). Then we have

$$\begin{aligned} A &= \int_{-2}^{2} [(y^2 + 1) - (y^4 - 2y^2 - 3)]\, dy \\ &= \int_{-2}^{2} [-y^4 + 3y^2 + 4]\, dy \\ &= \left[-\frac{1}{5}y^5 + y^3 + 4y \right]_{-2}^{2} \\ &= \frac{96}{5}. \end{aligned}$$