

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1

MATHEMATICS 1001-003

WINTER 2025

SOLUTIONS

- [5] 1. (a) Let $u = 13 - x^2$ so $du = -2x dx$ and $-\frac{1}{2} du = x dx$. The integral becomes

$$\begin{aligned}\int \frac{4x}{\sqrt{13-x^2}} dx &= 4 \int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{2} du\right) \\ &= -2 \int u^{-\frac{1}{2}} du \\ &= -2 \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right] + C \\ &= -4\sqrt{13-x^2} + C.\end{aligned}$$

- [4] (b) We can expand the integrand to obtain

$$\begin{aligned}\int \sqrt{4x}(13-x^2) dx &= \int (26x^{\frac{1}{2}} - 2x^{\frac{5}{2}}) dx \\ &= 26 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \cdot \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + C \\ &= \frac{52}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{7}{2}} + C.\end{aligned}$$

- [6] (c) First we complete the square:

$$\begin{aligned}13 + 4x - x^2 &= -(x^2 - 4x - 13) \\ &= -[(x^2 - 4x + 4) - 13 - 4] \\ &= -[(x-2)^2 - 17] \\ &= 17 - (x-2)^2.\end{aligned}$$

Thus the integral can be rewritten as

$$\int \frac{1}{\sqrt{13+4x-x^2}} dx = \int \frac{1}{\sqrt{17-(x-2)^2}} dx.$$

Now let $u = x - 2$ so $du = dx$ and we have

$$\begin{aligned}\int \frac{1}{\sqrt{13+4x-x^2}} dx &= \int \frac{1}{\sqrt{17-u}} du \\ &= \arcsin\left(\frac{u}{\sqrt{17}}\right) + C \\ &= \arcsin\left(\frac{\sqrt{17}}{17}(x-2)\right) + C.\end{aligned}$$

[25] 2. (a) Let $u = \ln(x)$ so $du = \frac{1}{x} dx$. The integral becomes

$$\begin{aligned} \int \frac{1}{x[\ln^2(x) + 4]} dx &= \int \frac{1}{u^2 + 4} du \\ &= \frac{1}{2} \arctan\left(\frac{u}{2}\right) + C \\ &= \frac{1}{2} \arctan\left(\frac{\ln(x)}{2}\right) + C. \end{aligned}$$

(b) We use integration by parts with $w = \arcsin(x)$ so $dw = \frac{1}{\sqrt{1-x^2}} dx$, and $dv = dx$ so $v = x$. The integral becomes

$$\int \arcsin(x) dx = x \arcsin(x) - \int \frac{x}{\sqrt{1-x^2}} dx.$$

Now let $u = 1 - x^2$ so $du = -2x dx$ and $-\frac{1}{2} du = x dx$. This yields

$$\begin{aligned} \int \arcsin(x) dx &= x \arcsin(x) + \frac{1}{2} \int u^{-\frac{1}{2}} du \\ &= x \arcsin(x) + \frac{1}{2} \left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}} \right] + C \\ &= x \arcsin(x) + \sqrt{1-x^2} + C. \end{aligned}$$

(c) Using long division of polynomials, we have

$$\begin{array}{r} 3x^2 + 2x \\ 2x - 1 \overline{) 6x^3 + x^2 - 2x - 8} \\ \underline{6x^3 - 3x^2} \\ 4x^2 - 2x - 8 \\ \underline{4x^2 - 2x} \\ -8 \end{array}$$

Thus we can write the integral as

$$\begin{aligned} \int \frac{6x^3 + x^2 - 2x - 8}{2x - 1} dx &= \int \left(3x^2 + 2x - \frac{8}{2x - 1} \right) dx \\ &= 3 \cdot \frac{x^3}{3} + 2 \cdot \frac{x^2}{2} - 8 \cdot \frac{1}{2} \ln|2x - 1| + C \\ &= x^3 + x^2 - 4 \ln|2x - 1| + C. \end{aligned}$$

(d) We rewrite the integrand as

$$\begin{aligned}\int \frac{\sin(3x) + 1}{\cos(3x)} dx &= \int \left[\frac{\sin(3x)}{\cos(3x)} + \frac{1}{\cos(3x)} \right] dx \\ &= \int [\tan(3x) + \sec(3x)] dx \\ &= \frac{1}{3} \ln|\sec(3x)| + \frac{1}{3} \ln|\sec(3x) + \tan(3x)| + C.\end{aligned}$$

(e) Observe that

$$\int x^5 \cos(x^3) dx = \int x^3 \cos(x^3) \cdot x^2 dx.$$

So let $u = x^3$ so $du = 3x^2 dx$ and $\frac{1}{3} du = x^2 dx$. The integral becomes

$$\int x^5 \cos(x^3) dx = \frac{1}{3} \int u \cos(u) du.$$

Now we use integration by parts. Let $w = u$ so $dw = du$, and let $dv = \cos(u) du$ so $v = \sin(u)$. Then

$$\begin{aligned}\int x^5 \cos(x^3) dx &= \frac{1}{3} \left[u \sin(u) - \int \sin(u) du \right] \\ &= \frac{1}{3} [u \sin(u) + \cos(u)] + C \\ &= \frac{1}{3} x^3 \sin(x^3) + \frac{1}{3} \cos(x^3) + C.\end{aligned}$$

(f) We use integration by parts with $w = x^2$ so $dw = 2x dx$, and $dv = e^{7x} dx$ so $v = \frac{1}{7}e^{7x}$. Hence

$$\int x^2 e^{7x} dx = \frac{1}{7} x^2 e^{7x} - \frac{2}{7} \int x e^{7x} dx.$$

Now we use integration by parts again, this time with $w = x$ so $dw = dx$, and $dv = e^{7x} dx$ so $v = \frac{1}{7}e^{7x}$. We obtain

$$\begin{aligned}\int x^2 e^{7x} dx &= \frac{1}{7} x^2 e^{7x} - \frac{2}{7} \left[\frac{1}{7} x e^{7x} - \frac{1}{7} \int e^{7x} dx \right] \\ &= \frac{1}{7} x^2 e^{7x} - \frac{2}{49} x e^{7x} + \frac{2}{49} \int e^{7x} dx \\ &= \frac{1}{7} x^2 e^{7x} - \frac{2}{49} x e^{7x} + \frac{2}{49} \left[\frac{1}{7} e^{7x} \right] + C \\ &= \frac{1}{7} x^2 e^{7x} - \frac{2}{49} x e^{7x} + \frac{2}{343} e^{7x} + C.\end{aligned}$$