MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

TEST 1 | **MATHEMATICS 1001-001** | WINTER 2025

SOLUTIONS

[6] 1. (a) First we complete the square:

$$\begin{aligned} 13 + 4x - x^2 &= -[x^2 - 4x - 13] \\ &= -[(x^2 - 4x + 4) - 13 - 4] \\ &= -[(x-2)^2 - 17] \\ &= 17 - (x-2)^2. \end{aligned}$$

Thus the integral can be rewritten as

$$\int \frac{1}{\sqrt{13 + 4x - x^2}}\,dx = \int \frac{1}{\sqrt{17 - (x-2)^2}}\,dx.$$

Now let $u = x - 2$ so $du = dx$ and we have

$$\begin{aligned} \int \frac{1}{\sqrt{13 + 4x - x^2}}\,dx &= \int \frac{1}{\sqrt{17 - u}}\,du \\ &= \arcsin\left(\frac{u}{\sqrt{17}}\right) + C \\ &= \arcsin\left(\frac{\sqrt{17}}{17}(x-2)\right) + C. \end{aligned}$$

[5] (b) Let $u = 13 - x^2$ so $du = -2x\,dx$ and $-\frac{1}{2}\,du = x\,dx$. The integral becomes

$$\begin{aligned} \int \frac{4x}{\sqrt{13 - x^2}}\,dx &= 4\int \frac{1}{\sqrt{u}} \cdot \left(-\frac{1}{2}\,du\right) \\ &= -2\int u^{-\frac{1}{2}}\,du \\ &= -2\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C \\ &= -4\sqrt{13 - x^2} + C. \end{aligned}$$

[4] (c) We can expand the integrand to obtain

$$\begin{aligned} \int \sqrt{4x}(13 - x^2)\,dx &= \int (26x^{\frac{1}{2}} - 2x^{\frac{5}{2}})\,dx \\ &= 26 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \cdot \frac{x^{\frac{7}{2}}}{\frac{7}{2}} + C \\ &= \frac{52}{3}x^{\frac{3}{2}} - \frac{4}{7}x^{\frac{7}{2}} + C. \end{aligned}$$

[25] 2. (a) Observe that

$$\int x^5 \cos(x^3)\, dx = \int x^3 \cos(x^3) \cdot x^2\, dx.$$

So let $u = x^3$ so $du = 3x^2\, dx$ and $\frac{1}{3}\, du = x^2\, dx$. The integral becomes

$$\int x^5 \cos(x^3)\, dx = \frac{1}{3} \int u \cos(u)\, du.$$

Now we use integration by parts. Let $w = u$ so $dw = du$, and let $dv = \cos(u)\, du$ so $v = \sin(u)$. Then

$$\begin{aligned} \int x^5 \cos(x^3)\, dx &= \frac{1}{3}\left[u \sin(u) - \int \sin(u)\, du\right] \\ &= \frac{1}{3}[u \sin(u) + \cos(u)] + C \\ &= \frac{1}{3} x^3 \sin(x^3) + \frac{1}{3} \cos(x^3) + C. \end{aligned}$$

(b) Let $u = \ln(x)$ so $du = \frac{1}{x}\, dx$. The integral becomes

$$\begin{aligned} \int \frac{1}{x[\ln^2(x) + 4]}\, dx &= \int \frac{1}{u^2 + 4}\, du \\ &= \frac{1}{2} \arctan\left(\frac{u}{2}\right) + C \\ &= \frac{1}{2} \arctan\left(\frac{\ln(x)}{2}\right) + C. \end{aligned}$$

(c) We use integration by parts with $w = x^2$ so $dw = 2x\, dx$, and $dv = e^{7x}\, dx$ so $v = \frac{1}{7} e^{7x}$. Hence

$$\int x^2 e^{7x}\, dx = \frac{1}{7} x^2 e^{7x} - \frac{2}{7} \int x e^{7x}\, dx.$$

Now we use integration by parts again, this time with $w = x$ so $dw = dx$, and $dv = e^{7x}\, dx$ so $v = \frac{1}{7} e^{7x}$. We obtain

$$\begin{aligned} \int x^2 e^{7x}\, dx &= \frac{1}{7} x^2 e^{7x} - \frac{2}{7}\left[\frac{1}{7} x e^{7x} - \frac{1}{7} \int e^{7x}\, dx\right] \\ &= \frac{1}{7} x^2 e^{7x} - \frac{2}{49} x e^{7x} + \frac{2}{49} \int e^{7x}\, dx \\ &= \frac{1}{7} x^2 e^{7x} - \frac{2}{49} x e^{7x} + \frac{2}{49}\left[\frac{1}{7} e^{7x}\right] + C \\ &= \frac{1}{7} x^2 e^{7x} - \frac{2}{49} x e^{7x} + \frac{2}{343} e^{7x} + C. \end{aligned}$$

(d) We rewrite the integrand as

$$\begin{aligned}\int \frac{\sin(3x)+1}{\cos(3x)}\,dx &= \int \left[\frac{\sin(3x)}{\cos(3x)} + \frac{1}{\cos(3x)}\right] dx \\ &= \int [\tan(3x)+\sec(3x)]\,dx \\ &= \frac{1}{3}\ln|\sec(3x)| + \frac{1}{3}\ln|\sec(3x)+\tan(3x)| + C.\end{aligned}$$

(e) We use integration by parts with $w = \arcsin(x)$ so $dw = \frac{1}{\sqrt{1-x^2}}\,dx$, and $dv = dx$ so $v = x$. The integral becomes

$$\int \arcsin(x)\,dx = x\arcsin(x) - \int \frac{x}{\sqrt{1-x^2}}\,dx.$$

Now let $u = 1-x^2$ so $du = -2x\,dx$ and $-\frac{1}{2}\,dx = x\,dx$. This yields

$$\begin{aligned}\int \arcsin(x)\,dx &= x\arcsin(x) + \frac{1}{2}\int u^{-\frac{1}{2}}\,du \\ &= x\arcsin(x) + \frac{1}{2}\left[\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right] + C \\ &= x\arcsin(x) + \sqrt{1-x^2} + C.\end{aligned}$$

(f) Using long division of polynomials, we have

$$\begin{array}{r|l} & 3x^2 + 2x \\ \hline 2x-1 & 6x^3 + x^2 - 2x - 8 \\ & 6x^3 - 3x^2 \\ \hline & \quad 4x^2 - 2x - 8 \\ & \quad 4x^2 - 2x \\ \hline & \qquad -8 \end{array}$$

Thus we can write the integral as

$$\begin{aligned}\int \frac{6x^3+x^2-2x-8}{2x-1}\,dx &= \int \left(3x^2 + 2x - \frac{8}{2x-1}\right) dx \\ &= 3\cdot\frac{x^3}{3} + 2\cdot\frac{x^2}{2} - 8\cdot\frac{1}{2}\ln|2x-1| + C \\ &= x^3 + x^2 - 4\ln|2x-1| + C.\end{aligned}$$