

MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 1.2

Math 1001 Worksheet

WINTER 2023

SOLUTIONS

1. (a) Let $u = 1 - x^3$ so

$$du = -3x^2 dx \implies -\frac{1}{3} du = x^2 dx.$$

The integral becomes

$$\int x^2(1 - x^3)^6 dx = -\frac{1}{3} \int u^6 du = -\frac{1}{3} \left[\frac{1}{7} u^7 \right] + C = -\frac{1}{21} (1 - x^3)^7 + C.$$

- (b) Let $u = 4t^2 + 5$ so

$$du = 8t dt \implies \frac{1}{8} du = t dt.$$

The integral becomes

$$\int \frac{t}{\sqrt[3]{4t^2 + 5}} dt = \frac{1}{8} \int \frac{du}{\sqrt[3]{u}} = \frac{1}{8} \int u^{-\frac{1}{3}} du = \frac{1}{8} \left[\frac{3}{2} u^{\frac{2}{3}} \right] + C = \frac{3}{16} (4t^2 + 5)^{\frac{2}{3}} + C.$$

- (c) Let $u = e^x + 3$ so $du = e^x dx$. The integral becomes

$$\int \frac{e^x}{e^x + 3} dx = \int \frac{du}{u} = \ln|u| + C = \ln|e^x + 3| + C.$$

- (d) Let $u = \cot(x)$ so

$$du = -\csc^2(x) dx \implies -du = \csc^2(x) dx.$$

The integral becomes

$$\int \frac{1}{2} \cot^3(x) \csc^2(x) dx = -\frac{1}{2} \int u^3 du = -\frac{1}{2} \left[\frac{1}{4} u^4 \right] + C = -\frac{1}{8} \cot^4(x) + C.$$

- (e) Let $u = \sin(4\theta^2)$ so

$$du = 8\theta \cos(4\theta^2) d\theta \implies \frac{1}{8} d\theta = \theta \cos(4\theta^2) d\theta.$$

The integral becomes

$$\int \theta \sin(4\theta^2) \cos(4\theta^2) d\theta = \frac{1}{8} \int u du = \frac{1}{8} \left[\frac{1}{2} u^2 \right] + C = \frac{1}{16} \sin^2(4\theta^2) + C.$$

(f) Let $u = \sqrt{x}$ so

$$du = \frac{1}{2\sqrt{x}} dx \implies 2 du = \frac{1}{\sqrt{x}} dx.$$

The integral becomes

$$\int \frac{\cot(\sqrt{x})}{\sqrt{x}} dx = 2 \int \cot(u) du = 2 \ln|\sin(u)| + C = 2 \ln|\sin(\sqrt{x})| + C.$$

(g) Let $u = x^3 - x$ so $du = 3x^2 - 1$. The integral becomes

$$\int \frac{6x^2 - 2}{x^3 - x} dx = \int \frac{2(3x^2 - 1)}{x^3 - x} dx = 2 \int \frac{du}{u} = 2 \ln|u| + C = 2 \ln|x^3 - x| + C.$$

(h) Using long division of polynomials, we have

$$\begin{array}{r} x^2 + 2x + 1 \\ x - 2 \overline{) x^3 - 3x + 1} \\ \underline{x^3 - 2x^2} \\ 2x^2 - 3x + 1 \\ \underline{2x^2 - 4x} \\ x + 1 \\ \underline{x - 2} \\ 3 \end{array}$$

so

$$\frac{x^3 - 3x + 1}{x - 2} = x^2 + 2x + 1 + \frac{3}{x - 2}.$$

Then

$$\int \frac{x^3 - 3x + 1}{x - 2} dx = \frac{1}{3}x^3 + x^2 + x + 3 \ln|x - 2| + C.$$

(i) Let $u = \cos^2(x)$ so

$$du = -2 \sin(x) \cos(x) dx \implies -\frac{1}{2} du = \sin(x) \cos(x) dx.$$

The integral becomes

$$\int e^{\cos^2(x)} \sin(x) \cos(x) dx = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + C = -\frac{1}{2} e^{\cos^2(x)} + C.$$

(j) Let $u = \ln(x)$ so $du = \frac{dx}{x}$. The integral becomes

$$\int \frac{1}{x\sqrt{\ln(x)}} dx = \frac{1}{\sqrt{u}} du = \int u^{-\frac{1}{2}} du = 2u^{\frac{1}{2}} + C = 2\sqrt{\ln(x)} + C.$$

(k) Let $u = 5 - x$ so

$$du = -dx \implies -du = dx.$$

Also, $x = 5 - u$ so $x - 2 = 3 - u$. The integral becomes

$$\begin{aligned} \int (x - 2)\sqrt{5 - x} dx &= - \int (3 - u)\sqrt{u} du \\ &= -3 \int u^{\frac{1}{2}} du + \int u^{\frac{3}{2}} du \\ &= -3 \left[\frac{2}{3} u^{\frac{3}{2}} \right] + \frac{2}{5} u^{\frac{5}{2}} + C \\ &= -2(5 - x)^{\frac{3}{2}} + \frac{2}{5}(5 - x)^{\frac{5}{2}} + C. \end{aligned}$$

(l) Let $u = x^2 + 8$ so

$$du = 2x dx \implies \frac{1}{2} du = x dx.$$

Also note that $x^2 = u - 8$. The integral becomes

$$\begin{aligned} \int x^3(x^2 + 8)^4 dx &= \int x^2(x^2 + 8)^4 x dx \\ &= \frac{1}{2} \int (u - 8)u^4 du \\ &= \frac{1}{2} \int u^5 du - 4 \int u^4 du \\ &= \frac{1}{2} \left[\frac{1}{6} u^6 \right] - 4 \left[\frac{1}{5} u^5 \right] + C \\ &= \frac{1}{12}(x^2 + 8)^6 - \frac{4}{5}(x^2 + 8)^5 + C. \end{aligned}$$