eg ① $\int \frac{1}{\sqrt{9-x^2}}\,dx$ $\qquad \int \frac{1}{\sqrt{k^2-x^2}}\,dx = \arcsin\left(\frac{x}{k}\right)+C$

$$\boxed{= \arcsin\left(\frac{x}{3}\right)+C}$$

② $\int \frac{8x}{\sqrt{9-x^2}}\,dx$ $\qquad$ Let $u = 9-x^2$

$du = -2x\,dx \rightarrow -\frac{1}{2}du = x\,dx$

$$= 8\int \frac{1}{\sqrt{u}}\cdot\left(-\frac{1}{2}du\right)$$

$$= -4\int u^{-1/2}\,du$$

$$= -4\left[\frac{u^{1/2}}{1/2}\right]+C = -8\sqrt{9-x^2}+C$$

③ $\int \frac{1}{\sqrt{9-8x-x^2}}\,dx$

We complete the square:

$$9-8x-x^2 = -\left[x^2+8x-9\right]$$

$$= -\left[(x^2+8x)-9\right]$$

$$= -\left[(x^2+8x+16)-9-16\right]$$

$$= -\left[(x+4)^2-25\right]$$

$$= 25-(x+4)^2$$

Now we have

$$\int \frac{1}{\sqrt{9-8x-x^2}}\,dx = \int \frac{1}{\sqrt{25-(x+4)^2}}\,dx \qquad \begin{array}{l} u = x+4 \\ du = dx \end{array}$$

$$= \int \frac{1}{\sqrt{25-u^2}}\,du$$

$$= \arcsin\left(\frac{u}{5}\right) + C$$

$$\boxed{= \arcsin\left(\frac{x+4}{5}\right) + C}$$

(4) $\int x \cos(9-8x)\,dx$

We use integration by parts with

$w = x \qquad dw = dx$

$dv = \cos(9-8x)\,dx \qquad V = \dfrac{\sin(9-8x)}{-8}$

Then $\int x\cos(9-8x)\,dx = -\frac{1}{8}x\sin(9-8x) + \frac{1}{8}\int \sin(9-8x)\,dx$

$$= -\frac{1}{8}x\sin(9-8x) + \frac{1}{8}\cdot\frac{-\cos(9-8x)}{-8} + C$$

$$\boxed{= -\frac{1}{8}x\sin(9-8x) + \frac{1}{64}\cos(9-8x) + C}$$