eg ① $\displaystyle\int \frac{8x^3-2x^2+32x-5}{2x^2+8}\,dx$

We rewrite the integrand using long division:

$$\begin{array}{r|l} & 4x-1 \\ 2x^2+8 & 8x^3-2x^2+32x-5 \\ & 8x^3 \qquad\quad +32x \\ \hline & \quad -2x^2 \qquad\quad -5 \\ & \quad -2x^2 \qquad\quad -8 \\ \hline & \qquad\qquad\qquad 3 \end{array}$$

We can rewrite the integral as

$$\int \frac{8x^3-2x^2+32x-5}{2x^2+8}\,dx$$

$$= \int \left(4x - 1 + \frac{3}{2x^2+8}\right) dx$$

$$= 4\left[\frac{x^2}{2}\right] - x + \frac{3}{2}\int \frac{1}{x^2+4}\,dx$$

$$= 2x^2 - x + \frac{3}{2}\cdot\frac{1}{2}\arctan\left(\frac{x}{2}\right) + C$$

$$\boxed{= 2x^2 - x + \frac{3}{4}\arctan\left(\frac{x}{2}\right) + C}$$

② $\int \sin(2x)\sin(x)\,dx$

We use integration by parts with

$$w = \sin(2x) \qquad dw = 2\cos(2x)\,dx$$

$$dv = \sin(x)\,dx \qquad v = -\cos(x)$$

so we have

$$\int \sin(2x)\sin(x)\,dx = -\sin(2x)\cos(x) + 2\int \cos(2x)\cos(x)\,dx$$

We try integration by parts again with

$$w = \cos(2x) \qquad dw = -2\sin(2x)\,dx$$

$$dv = \cos(x)\,dx \qquad v = \sin(x)$$

This gives

$$\int \sin(2x)\sin(x)\,dx = -\sin(2x)\cos(x) + 2\left[\cos(2x)\sin(x) + 2\int \sin(2x)\sin(x)\,dx\right]$$

$$= -\sin(2x)\cos(x) + 2\cos(2x)\sin(x) + 4\int \sin(2x)\sin(x)\,dx$$

$$-3\int \sin(2x)\sin(x)\,dx = -\sin(2x)\cos(x) + 2\cos(2x)\sin(x)$$

$$\int \sin(2x)\sin(x)\,dx \boxed{= \frac{1}{3}\sin(2x)\cos(x) - \frac{2}{3}\cos(2x)\sin(x) + C}$$

This can be rewritten using the double angle formulas:

$$\int \sin(2x)\sin(x)\,dx = \frac{2}{3}\sin(x)\cos^2(x) - \frac{2}{3}\left[\cos^2(x) - \sin^2(x)\right]\sin(x) + C$$

$$= \frac{2}{3}\sin^3(x) + C$$

as in Tutorial #2.