

MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

SECTION 4.3

Math 1001 Worksheet

WINTER 2024

For practice only. Not to be submitted.

1. A colony of 50 rabbits is relocated to an island where it grows logistically, modelled by the differential equation

$$\frac{dy}{dt} = \frac{1}{5}y \left(1 - \frac{1}{1000}y \right),$$

where t is measured in years.

- (a) Determine the carrying capacity of the rabbit population.
 - (b) Determine the size of the population after 5 years.
 - (c) To the nearest year, determine when the population reaches 500 rabbits.
2. Two populations of fruit flies are found to grow according to the same logistic differential equation:

$$\frac{dy}{dt} = \frac{2}{9}y \left(1 - \frac{1}{800}y \right).$$

One population initially includes 30 fruit flies, while the other population initially includes 3000 fruit flies. Determine what happens to the size of each population as $t \rightarrow \infty$.

3. The town of Barovia is founded with an initial population of 50 villagers. It is located in a fertile valley with abundant space, so the population can be expected to follow the natural growth model, and reach about 51,200 villagers after 10 years. However, the building of the town awakens the vampire Count Strahd, who demands that a villager be surrendered to his appetites every fortnight (that is, every 2 weeks), lest he lay waste to the entire population.
- (a) With this constant harvesting in place, and assuming that there are 52 weeks in a Barovian year, what population does Barovia actually achieve after 10 years?
 - (b) Would the population of Barovia survive if Count Strahd instead demands a villager every week?
4. Unlike constant harvesting, *effort harvesting* assumes that the amount of a population that is regularly harvested is proportional to the current size of the population.
- (a) If the population y would otherwise follow the natural growth model, express a differential equation that models effort harvesting. Use h as the constant of proportionality that determines the amount of harvesting.
 - (b) Solve the differential equation, assuming $y(0) = y_0$.
 - (c) What condition on h is necessary for the population to survive when it experiences effort harvesting?