MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Assignment 5	**Mathematics 1001**	Winter 2024

SOLUTIONS

[4] 1. First we have to rewrite the given integral as the sum of two integrals:

$$f(x) = \int_x^0 \csc(t^3)\,dt + \int_0^{x^2} \csc(t^3)\,dt = -\int_0^x \csc(t^3)\,dt + \int_0^{x^2} \csc(t^3)\,dt.$$

Now we can apply the Fundamental Theorem of Calculus (twice), with the Chain Rule necessary for the second use:

$$f'(x) = -\csc(x^3) + \csc((x^2)^3)\cdot[x^2]' = -\csc(x^3) + 2x\csc(x^6).$$

[4] 2. (a) We require integration by parts, with $w = \ln(x)$ so $dw = \frac{1}{x}\,dx$, and $dv = x\,dx$ so $v = \frac{1}{2}x^2$. Then

$$\begin{aligned}\int_1^3 x\ln(x)\,dx &= \left[\frac{1}{2}x^2\ln(x)\right]_1^3 - \frac{1}{2}\int_1^3 x\,dx \\ &= \left[\frac{1}{2}x^2\ln(x) - \frac{1}{4}x^2\right]_1^3 \\ &= \left[\frac{1}{2}(3^2)\ln(3) - \frac{1}{4}(3^2)\right] - \left[\frac{1}{2}(1^2)\ln(1) - \frac{1}{4}(1^2)\right] \\ &= \frac{9}{2}\ln(3) - 2.\end{aligned}$$

[5] (b) Let $u = \tan(x)$ so $du = \sec^2(x)\,dx$. When $x = 0$, $u = 0$. When $x = \frac{\pi}{6}$, $u = \frac{\sqrt{3}}{3}$. Thus the integral becomes

$$\begin{aligned}\int_0^{\frac{\pi}{6}} \tan^5(x)\sec^2(x)\,dx &= \int_0^{\frac{\sqrt{3}}{3}} u^5\,du \\ &= \left[\frac{1}{6}u^6\right]_0^{\frac{\sqrt{3}}{3}} \\ &= \frac{1}{6}\left[\left(\frac{\sqrt{3}}{3}\right)^6 - 0^6\right] \\ &= \frac{1}{162}.\end{aligned}$$

[4] (c) First observe that $x^2 - 9 = 0$ when $x = \pm 3$. When $-3 < x < 3$, $x^2 - 9 < 0$. Otherwise, $x^2 - 9 > 0$. This means that we can write

$$|x^2 - 9| = \begin{cases} x^2 - 9 & \text{if } x \geq 3 \text{ or } x \leq -3 \\ -(x^2 - 9) & \text{if } -3 < x < 3. \end{cases}$$

Now we can use the additive property of definite integrals to write

$$\begin{aligned}
\int_{-5}^{4} |x^2 - 9| \, dx &= \int_{-5}^{-3} |x^2 - 9| \, dx + \int_{-3}^{3} |x^2 - 9| \, dx + \int_{3}^{4} |x^2 - 9| \, dx \\
&= \int_{-5}^{-3} (x^2 - 9) \, dx - \int_{-3}^{3} (x^2 - 9) \, dx + \int_{3}^{4} (x^2 - 9) \, dx \\
&= \left[\frac{1}{3}x^3 - 9x\right]_{-5}^{-3} - \left[\frac{1}{3}x^3 - 9x\right]_{-3}^{3} + \left[\frac{1}{3}x^3 - 9x\right]_{3}^{4} \\
&= \frac{44}{3} - (-36) + \frac{10}{3} \\
&= 54.
\end{aligned}$$

[3] 3. By the Fundamental Theorem, we simply have

$$\begin{aligned}
\int_{0}^{2} (x^3 - 2x + 4) \, dx &= \left[\frac{1}{4}x^4 - x^2 + 4x\right]_{0}^{2} \\
&= 4 - 4 + 8 \\
&= 8.
\end{aligned}$$