MEMORIAL UNIVERSITY OF NEWFOUNDLAND

DEPARTMENT OF MATHEMATICS AND STATISTICS

Test 2	**MATHEMATICS 1001-003**	Winter 2026

SOLUTIONS

[7] 1. (a) We use a regular partition with subintervals of width

$$\Delta x = \frac{1-(-1)}{n} = \frac{2}{n}.$$

We choose the sample point

$$x_i^* = x_i = -1 + i\Delta x = \frac{2i}{n} - 1.$$

Thus

$$\begin{aligned} f(x_i^*) &= \left[3\left((\frac{2i}{n} - 1\right) - 1\right]^2 \\ &= \left[\frac{6i}{n} - 4\right]^2 \\ &= \frac{36i^2}{n^2} - \frac{48i}{n} + 16. \end{aligned}$$

Now we can write

$$\begin{aligned} \int_{-1}^{1} (3x-1)^2)\,dx &= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x \\ &= \lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{36i^2}{n^2} - \frac{48i}{n} + 16\right) \cdot \frac{2}{n} \\ &= \lim_{n\to\infty} \left[\frac{72}{n^3}\sum_{i=1}^{n} i^2 - \frac{96}{n^2}\sum_{i=1}^{n} i + \frac{32}{n}\sum_{i=1}^{n} 1\right] \\ &= \lim_{n\to\infty} \left[\frac{72}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} - \frac{96}{n^2} \cdot \frac{n(n+1)}{2} + \frac{32}{n} \cdot n\right] \\ &= \lim_{n\to\infty} \left[\frac{12(n+1)(2n+1)}{n^2} - \frac{48(n+1)}{n} + 32\right] \\ &= 24 - 48 + 32 \\ &= 8. \end{aligned}$$

[3] (b) We have

$$\begin{aligned}\int_{-1}^{1}(3x-1)^2\,dx &= \int_{-1}^{1}(9x^2-6x+1)\,dx\\ &= \left[9\cdot\frac{x^3}{3}-6\cdot\frac{x^2}{2}+x\right]_{-1}^{1}\\ &= \left[3x^3-3x^2+x\right]_{-1}^{1}\\ &= (3-3+1)-(-3-3-1)\\ &= 8.\end{aligned}$$

[6] 2. (a) First we rewrite the integral as

$$\begin{aligned}\int_{0}^{\frac{\pi}{2}}\sin^2(x)\cos^3(x)\,dx &= \int_{0}^{\frac{\pi}{2}}\sin^2(x)\cos^2(x)\cdot\cos(x)\,dx\\ &= \int_{0}^{\frac{\pi}{2}}\sin^2(x)[1-\sin^2(x)]\cdot\cos(x)\,dx.\end{aligned}$$

Now we let $u=\sin(x)$ so $du=\cos(x)\,dx$. When $x=0$, $u=\sin(0)=0$. When $x=\frac{\pi}{2}$, $u=\sin\left(\frac{\pi}{2}\right)=1$. Thus the integral becomes

$$\begin{aligned}\int_{0}^{\frac{\pi}{2}}\sin^2(x)\cos^3(x)\,dx &= \int_{0}^{1}u^2(1-u^2)\,du\\ &= \int_{0}^{1}(u^2-u^4)\,du\\ &= \left[\frac{u^3}{3}-\frac{u^5}{5}\right]_{0}^{1}\\ &= \left(\frac{1}{3}-\frac{1}{5}\right)-(0-0)\\ &= \frac{2}{15}.\end{aligned}$$

[5] (b) Using the Additive Interval Property,

$$\begin{aligned}
\int_0^4 f(x)\,dx &= \int_0^1 f(x)\,dx + \int_1^4 f(x)\,dx \\
&= \int_0^1 \frac{1}{x^2+1}\,dx + \int_1^4 \frac{1}{2\sqrt{x}}\,dx \\
&= \Big[\arctan(x)\Big]_0^1 + \Big[\sqrt{x}\Big]_1^4 \\
&= [\arctan(1) - \arctan(0)] + [\sqrt{4} - \sqrt{1}] \\
&= \frac{\pi}{4} + 1.
\end{aligned}$$

[6] (c) Since the integrand is a proper rational function, we use the method of partial fractions. The partial fraction decomposition has the form

$$\frac{6x^2+9}{x^2(x-3)} = \frac{A}{x-3} + \frac{Bx+C}{x^2}$$

$$6x^2+9 = Ax^2 + (Bx+C)(x-3).$$

When $x = 0$, we have $9 = -3C$ so $C = -3$. When $x = 3$, we have $63 = 9A$ so $A = 7$. Finally we can use, say, $x = 1$ and get

$$15 = A - 2(B+C) \implies 15 = 7 - 2B + 6 \implies 2 = -2B \implies B = -1.$$

Thus

$$\begin{aligned}
\int \frac{6x^2+9}{x^2(x-3)}\,dx &= \int \left(\frac{7}{x-3} + \frac{-x-3}{x^2}\right) dx \\
&= \int \left(\frac{7}{x-3} - \frac{1}{x} - \frac{3}{x^2}\right) dx \\
&= 7\ln|x-3| - \ln|x| + \frac{3}{x} + C.
\end{aligned}$$

[4] 3. First we write

$$g(x) = -\int_5^{x^3} \frac{\sin(t)}{t^2}\,dt.$$

Now we can apply the Fundamental Theorem of Calculus and the Chain Rule:

$$\begin{aligned}
g'(x) &= -\frac{\sin(x^3)}{(x^3)^2} \cdot [x^3]' \\
&= -\frac{\sin(x^3)}{x^6} \cdot 3x^2 \\
&= -\frac{3\sin(x^3)}{x^4}.
\end{aligned}$$

[9] 4. (a) The sketch of R is given in Figure 1. Note that the two curves intersect when

$$\begin{aligned} 2x &= 5 - 3x \\ 5x &= 5 \\ x &= 1. \end{aligned}$$

By substitution back into either function, we see that this is the point $(1, 2)$.

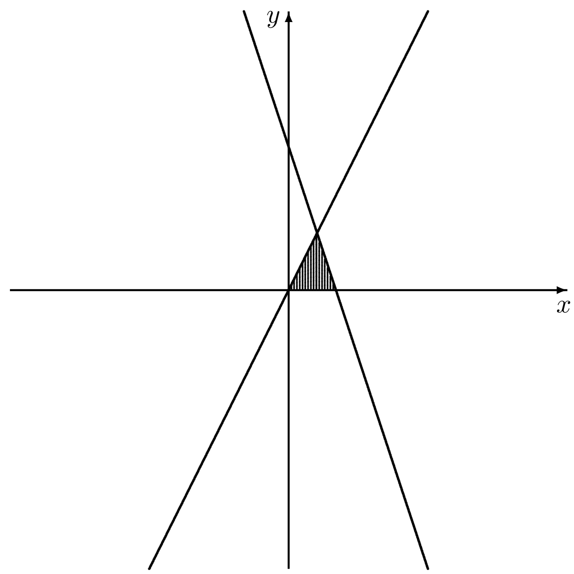

Figure 1: Question 4(a)

(b) The region is not vertically simple, but it can be split into vertically simple regions on the intervals $[0, 1]$ and $[1, \frac{5}{3}]$. From the graph, we can see that the curve $y = 0$ is always the bottom boundary curve, while $y = 2x$ is the top boundary curve on $[0, 1]$ and $y = 5 - 3x$ is the top boundary curve on $[1, \frac{5}{3}]$. Thus

$$\begin{aligned} A &= \int_0^1 (2x - 0)\, dx + \int_1^{\frac{5}{3}} [(5 - 3x) - 0]\, dx \\ &= \int_0^1 2x\, dx + \int_1^{\frac{5}{3}} (5 - 3x)\, dx. \end{aligned}$$

(c) The region is horizontally simple. The function $y = 2x$ can be written $x = \frac{1}{2}y$ (the lefthand boundary curve) while $y = 5 - 3x$ becomes $3x - 5 - y$ and so $x = \frac{5}{3} - \frac{1}{3}y$ (the righthand boundary curve). Hence

$$\begin{aligned} A &= \int_0^2 \left[\left(\frac{5}{3} - \frac{1}{3}y \right) - \frac{1}{2}y \right] dy \\ &= \int_0^2 \left(\frac{5}{3} - \frac{5}{6}y \right) dy. \end{aligned}$$